DEVELOPMENT OF AN OPTIMIZATION PACKAGE : A COMPACT HEAT EXCHANGER APPLICATION

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MARCH, 1984

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A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

By

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to the

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
MARCH, 1984

DEDICATED

TO

MY PARENTS

AS A MARK OF AFFECTION

ACKNOWLEDGEMENTS

At the outset, I wish to express my deep sense of gratitude to Prof. H.C. Agrawal for his valuable guidance. I am highly grateful to him for his understanding, keen encouragement, kind help extended to me in carrying out this project.

It is impossible to fully appreciate in words the warmth and affection that my innumerable friends accorded to me throughout the period of my stay here.

I personally thank all the people who have associated with me in all dimensions and in all spaces.

- C. VIJAYA KUMAR

CERTIFICATE

Certified that the work entitled "DEVELOPMENT OF AN OPTIMIZATION PACKAGE : A COMPACT HEAT EXCHANGER APPLICATION" by C. VIJAYA KUMAR has been carried out under my supervision and has not been submitted elsewhere for a degree.

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ABSTRACT

A package of general and efficient computer code for solving optimization problems is presented in this thesis. Powerful unconstrained optimization methods based on the ideas of quadratic convergence and conjugate directions are used. Constrained optimization problems are solved, using the penalty functions and feasible directions method.

These programs are tested for convergence and efficiency by using typical test functions previously suggested in the open literature. Results of a comparative study of constrained optimization methods indicate that sequential penalty function methods, with a gradient-based search procedure, using finite difference gradient computations, perform well on most problems.

The capability of the package is well demonstrated by the design of a direct transfer air-to-water compact heat exchanger employing flat-finned tube surface, the objective being minimum pumping power and the constraints imposed on volume, dimensions of the exchanger and heat transfer. The ε-NTU method is used for evaluating the heat exchanger performance. Polynomial expressions are found to approximate the experimental data for the fluid transport properties and the friction and colburn factors for the heat transfer surface. This optimization study of the compact heat exchanger provides a design which has got much superior performance.

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CHAPTER 1

INTRODUCTION

In modern technology, it is becoming increasingly important for engineering designs of all types to offer the best possible performance. To achieve this task, the analyst must identify the variables which influence the behaviour of the system, judiciously select quantitative measures that appropriately describe the system performance and then make a detailed parametric study. Frequently, it is found that some of the desired objectives have conflicting trends. Also, in many applications it is not possible to keep the design variables constant as they may have a tendency to drift with time or fluctuate between some limits. For a reliable engineering design it is not adequate to just determine the best operating conditions. It is equally important to know how sensitive the optimum performance is, to changes in the system parameters. Another important consideration that must be kept in mind while making a design study is that the design requirements may change in future, and it would be very useful if a suitable alternate design can be predicted from the results of the current optimum. This would eliminate the need for making another extensive parametric study. This necessitates the introduction of a design tool that will permit the engineer to experiment with design variables and different performance criteria, to search for the best operating conditions and to identify the influence of the system parameters as the selected performance indices. It is also desirable to include some aids to show how the design specifications constrain the system performance.

Computer-aided design procedures are playing an important role in the design practice, making it possible to analyze a wide variety of complex problems. Recent advances in computer-graphics, real time manipulation capabilities along with the developments in optimization theory and interactive computation have made it possible to tap the design tools'potential. The present work is concerned with the development of a single package which includes several optimization methods. The package has been used for optimum design of a compact heat exchanger.

1.1 HEAT EXCHANGER OPTIMIZATION

Heat exchangers are designed for many different applications and hence may involve many different performance criteria. These criteria may be:

minimum initial cost, minimum initial and operating costs, minimum weight or material, minimum volums or heat transfer surface area, and so on.

When a single performance measure has been defined quantitatively and is to be minimized or maximized, it is called an "objective function" in a design optimization. A particular design may be subjected to certain " CONSTRAINTS" such as desired heat transfer rate, allowable pressure drop, limitations on height, width and/or length of the exchanger. A number of different surfaces could be incorporated in a specific design problem and there are many geometrical parameters that could be varied for each surface geometry. For an extended surface exchanger, the geometrical variables associated with a fin are the fin pitch, fin height, fin thickness, type of fin, and other variables associated with each fin-type. In addition, operating flows and temperatures could also be changed. Thus a large number of " design variables" are associated with a heat exchanger design. The question arises as to how one can effectively adjust these design variables within imposed constraints and come up with a design having optimum objective function. This is what we mean by the most " efficient" design.

A complete mathematical optimization of heat exchanger design is neither practical nor possible.

Many engineering judgements based on experience are involved in different stages of the design. However, once the general configuration and surfaces are selected for a specific application, an optimized heat exchanger design may be arrived at if the objective function and constraints can be expressed mathematically, and if all of the variables are automatically and systematically changed on some statistical or mathematical basis.

A large number of optimization (search) techniques are available in literature. The difficulty with the present state of optimization art is that while any single search technique may work well on many problems, no single technique is able to solve every problem. A package of optimization programs is a convenient approach to overcoming this difficulty. Well developed, the programs can be used interchangeably and a new search can be called in, where a previous one has quit thereby continuing or assuring progress toward optimum. In addition, when a package is used, various parts of the search procedure common to several optimization routines need be programmed only a single time.

1.2 OPTIMIZATION

Optimization techniques offer systematic approach for seeking the best operating conditions. A description of many of the more popular optimization algorithms is presented in [1, 2, 3]. Almost all design problems require either the maximization or the minimization of some parameter. This parameter is usually called the design objective function. For example the problem may call for a heat exchanger with a minimum volume. The expression for volume would be the design objective function. This is done by selecting values for certain variables. These variables are known as design or decision variables. For the design to be acceptable, it must satisfy certain constraints. For example, an air heater must be designed so that it will fit into a given space.

The general nonlinear constrained optimization problem can be written mathematically as:

Minimize $F(\bar{X})$ (1)

Subject to: $g_i(\bar{X}) \leq 0 \quad i = 1, 2,..,m$ (2)

$l_k(\bar{X}) = 0 \quad k = 1, 2,..,p$ (3)

$X_q^l \leq \bar{X}_q \leq X_q^u \quad q = 1,2,...,n$ (4)

Where the vector, $\bar{X}$, is the vector of " n " design variables, F is the objective function, g_i is a set of " m " in-equality constraints, l_k is a set of " p" equality constraints and the lower and upper bounds X_q^l , X_q^u are the side constraints imposed on the design variables. Side constraints could also be included in (2), but for better efficiency they are treated separately by some software experts. Equations (2) and (3) may be linear or nonlinear functions of the design variables. They may be explicit or implicit functions of $\bar{X}$, but must have continuous first derivatives. The basic solution procedure is to, systematically, search the design space by sequentially varying the design parameters along search directions that reduce the objective function. For example, starting at a point $\bar{X}_q$ defined by the existing values for the design variables, a new point $\bar{X}_{q+1}$ is determined as:

$$\bar{X}_{q+1} = \bar{X}_q + \alpha \bar{S}_q$$

$$F(\bar{X}_{q+1}) < F(\bar{X}_q)$$

Where $\bar{S}_q$ is the search direction, and α is the step size along that direction. The essence of nonlinear programming methods lies in determining the α which minimizes $F(\bar{X}_{q+1})$ by using a one-dimensional search technique. A new search direction is computed at $\bar{X}_{q+1}$. A minimum is obtained if it satisfies the accuracy criterion. There is no evidence that the resulting minimum is the global optimum, since relative minima may be present. The search procedure must be repeated using different starting points if this is a possibility. For problems based on real processes, luckily, the objective function is usually a well behaved function with a single extremum. Therefore, for most practical purposes, the use of standard numerical procedures that provide a local solution to the optimization problem is not a great disadvantage.

1.3 PREVIOUS WORK

Compact heat exchanger design is a complex task requiring the examination and optimization of a wide variety of heat transfer surfaces. Smith [4] has listed some typical advantages of direct cooling with air as compared to cooling with water in a shell-and-tube exchanger. Studies have shown that a poor choice of either the heat transfer surfaces or design parameters can more than double the costs chargeable to a heat exchanger.

The method of Bergles et al. [5] is primarily meant for comparing the performance of heat exchanger surfaces for some specified criteria by adjusting only two exchanger variables. Their method does not include any minimization technique but results show that a great improvement in heat exchanger performance can be made by proper selection of design parameters.

Fax and Mills [6] developed a method, using Lagrange multipliers to optimize heat exchanger design under specified constraints. This technique requires the objective function and the constraints expressed in explicit equation form and differentiable throughout the range of interest, the total number of constraints be less than the total number of variables, and all constraints be equality constraints. Obviously, the use of this method is restricted to a limited number of problems. Briggs and Evans [7] recommended a " logic search method" in which an experienced engineer makes use of selecting design variables based on his apriori knowledge to obtain an optimum design. This method is less scientific and can be tried only if the computer time and storage capacity are freely ensured. Wilde [8] has surveyed various optimization techniques applicable to heat exchanger design. He focussed on the geometric programming technique and other search methods which require explicit expressions for the constraints and the objective function. The geometric programming technique also requires that all of the constraints and the objective function must be expressed in a particular power law equation form. These methods have limited applications for a general heat exchanger optimization problem.

Palen et al. [9] proposed the Complex Method [1], for the heat exchanger optimization problem. They found a minimum shell-and-tube exchanger by varying six geometrical parameters. The Complex Method requires several feasible starting designs before optimization can be performed. Johnson et al. [10] coupled an existing shell-and-tube condensor design code with a constrained function minimization code to produce an automated marine condensor design program of vastly different complexity.

Fontein and Wassink [11] utilized the Simplex Method [1] and steepest descent method for optimizing a shell-and-tube exchanger. Shah et al. [12, 13] have initiated work on package approach lines to the optimization problems in compact heat exchangers. The package includes the numerical nonlinear programming techniques. Hedderich et al. [14] have also developed a massive code using the method of feasible directions and the augmented Lagrange multiplier method. It has been used for the design of an air-to-water finned-tube heat exchanger.

It is experienced that, although, there are many methods that have been presented for heat-exchanger optimization, each of the methods has its own limitations; none is completely general. Among the design procedures cited above, those which are applicable, to cross flow, air cooled compact heat exchangers are limited to the 120 individual surfaces found in the open literature [15] for the calculation of the air side heat-transfer coefficient and friction factor. Therefore, the designer is faced with choosing an optimum surface from a number of individual optimal designs calculated from one of the above methods.

1.4 PRESENT WORK

The work embodied in this thesis is an attempt in two directions:

(i) to develop a general purpose computer aided design procedure that assists the engineer in establishing a best design. This makes available to the user, numerous nonlinear programming techniques. The search parameters can be changed and the user can switch from one search technique to another. This flexibility, provided to the user, gives a feel for the various technique's applicability and to select the most suitable one to the problem in hand. The development of this computer package is described in the next chapter,

(ii) to use the above package for optimizing the design of a compact heat exchanger for some specific objective and constraints. A direct transfer, air-water, finned-tube heat exchanger for a gas turbine plant has been optimized. The objective is to minimize pumping power requirements of the heat exchanger which has a volume less than a specified value and for which the heat transfer rate is prescribed. The flow rates of the fluids and the linear dimensions of the exchanger comprise the design variables of the problem. Fixed bounds on these variables are also imposed as side constraints. Constraints take into a account the aesthetic value to get a realistic configuration for the exchanger.

The package implements the following multi-dimensional minimization schemes: univariate, steepest descent, conjugate directions, conjugate gradient, variable metric method [1,2,3]. These methods use either a quadratic interpolation procedure or golden section technique [1,2,3] for 1-D minimization. Constrained problems are solved using either penalty function or feasible directions [1,3] method. To check the reliability and flexibility of the proposed package, a number of problems available in literature have been solved and results compared and listed in Appendix-A. These problems differ fundamentally in the manner in which the objective functions and the constraints are computed in the function subroutine. These quantities are usually programmed in the user written function subroutine as follows:

1. Closed form algebraic expressions
2. Mostly algebra but requiring some iterations

The programs are simple to use, with the user having to provide a function subroutine and to specify a minimum number of variables and flags. The ensuing chapter deals with the package structure that has been developed.

CHAPTER 2

PACKAGE STRUCTURE

2.1 INTRODUCTION

A set of general programs have been developed which make available to the user, reliable mathematical programming routines suitable for optimizing a large class of problems. Various numerical techniques which have been utilized to structure this optimization package are briefly discussed in this chapter and summarized in Fig. 1. Flow diagrams for solving unconstrained and constrained optimization problems are given in Fig. 2. Complete details regarding the algorithms used, may be obtained from [1,2,3]. Appendix-A gives the details for using the programs with examples to illustrate the procedure and program-listings have been given in Appendix-B.

An interesting feature of the package is it's flexibility. New search methods can be easily introduced by making minor changes. This is made possible by having maximum interaction between the subroutines and dividing the procedure into the following four basic operations:

1. Selection of a search direction
2. Estimation of the range for minimum along the search direction

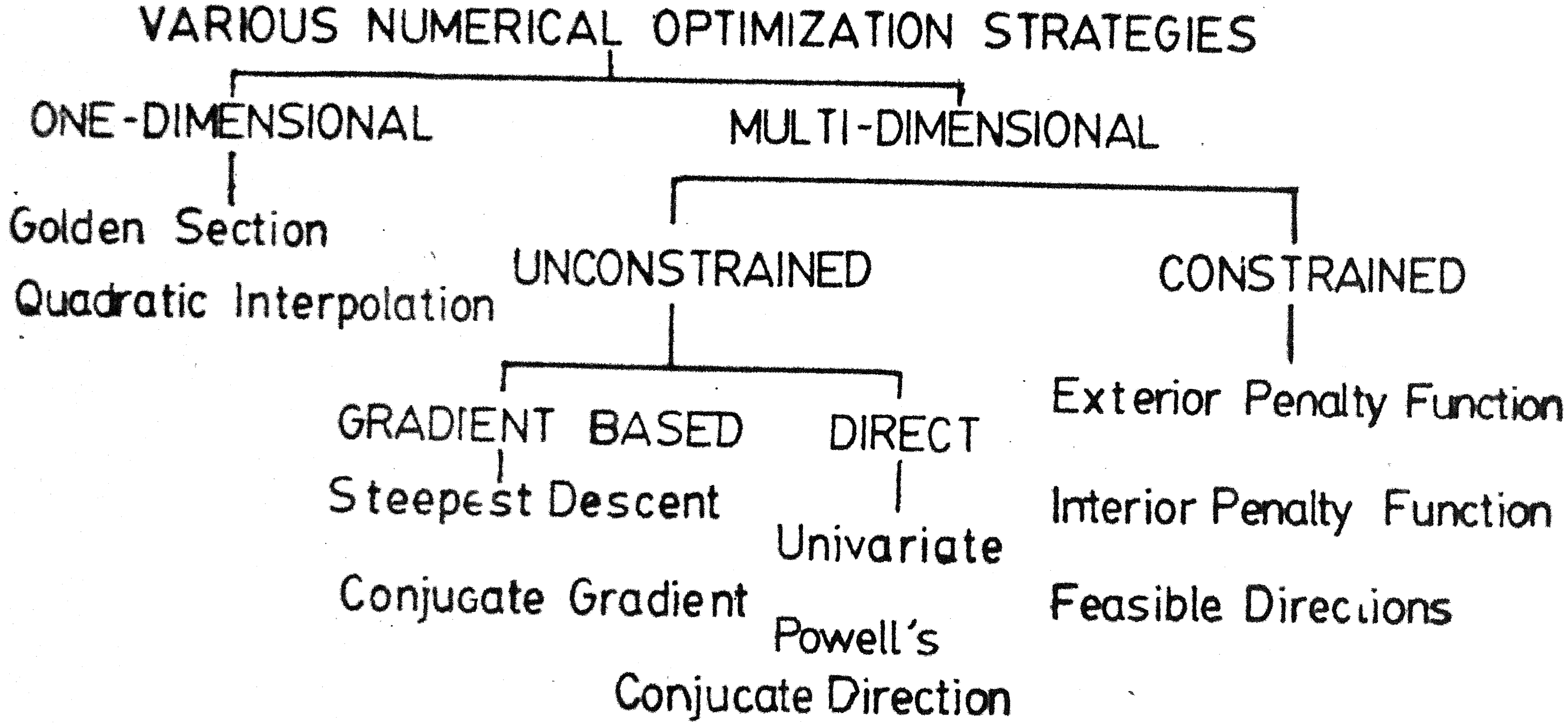


Fig1. Optimization Techniques

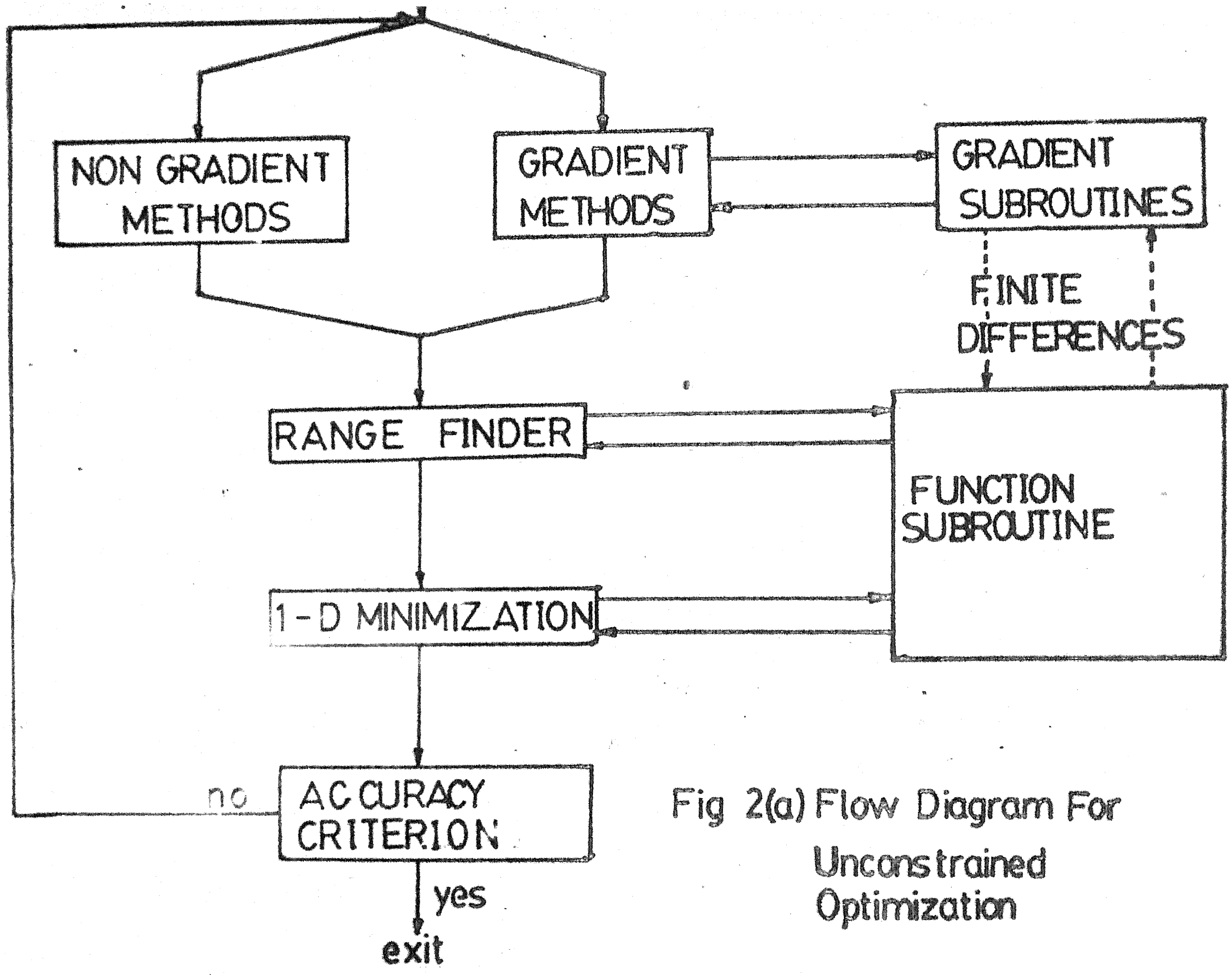


Fig 2(a) Flow Diagram For Unconstrained Optimization

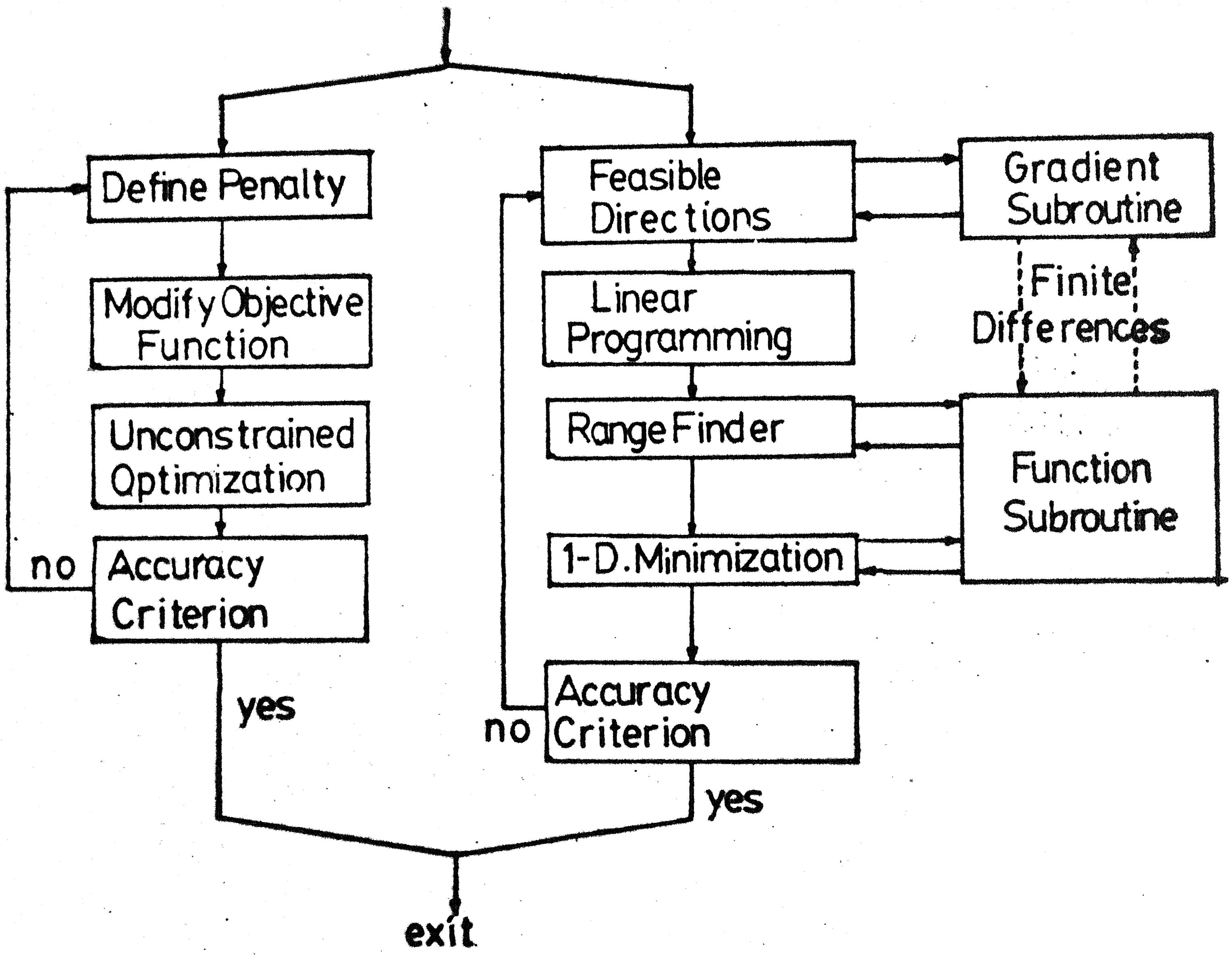


Fig 2(b). Flow chart for Constrained Optimization

3. One-dimensional minimization to locate the minimum along the search direction

4. Checking for convergence

The multidimensional methods differ primarily in the manner by which the search direction is determined. The effectiveness of the methods for computing a given search direction depends on the accuracy with which the minimum along previous search directions has been obtained. Thus, the one-dimensional minimization influences the stability and convergence of the multi-dimensional methods.

2.2 RANGE-FINDER: SUBROUTINE RANGE

Nonlinear programming methods essentially consist of a series of unidimensional minimizations. Numerical solution procedures for locating a minimum in a search direction require a prior knowledge of the limits between which the minimum exists. These limit points are determined by the range-finder by sequentially increasing the step-size along the search direction until three consecutive points are obtained, such that

$$\begin{aligned} F(\bar{X}_q + \alpha_1 \bar{S}_q) &> F(\bar{X}_q + \alpha_2 \bar{S}_q) \\ F(\bar{X}_q + \alpha_3 \bar{S}_q) &> F(\bar{X}_q + \alpha_2 \bar{S}_q) \end{aligned} \qquad (5)$$

where α_1 , α_2 , α_3 define three consecutive points along the search direction and $\bar{X}_q$ is the design vector the $\bar{S}_q$ is search direction. The Equation (5) guarantees that a minimum exists between α_1 and α_3.

The inputs to RANGE are $\bar{x}_q$, $\bar{s}_q$, initial stepsize α, maximum increment in stepsize and the number of iterations, permitted to determine α_1 , α_2 , and α_3. It is desirable to obtain a small value for $(\alpha_3 - \alpha_1)$ in a few iterations. If a step along $\bar{s}_q$ reduces F, α is doubled for the next iteration, and the values for α_1 and α_2 updated. The increment in α is not allowed to exceed the maximum specified value. When more iterations are required to estimate $(\alpha_3 - \alpha_1)$ than that specified, RANGE assumes the last value of α to be the location of the minimum along $\bar{s}_q$. While seeking α_2 , RANGE has the provision to reverse the search direction and reduce α, if an initial step along $\bar{s}_q$ increases F.

Since RANGE is used also by constrained optimization methods, some additional features have to be introduced to take care of constraints. When using the method of feasible directions, it is necessary to locate constraints that are violated. If the point $\bar{x}_{p+1}$ violates constraint "g", RANGE repetitively uses a linear interpolation scheme to find α' , where

$$\begin{aligned} \alpha' &= \frac{\alpha_p \, g(\bar{x}_{p+1}) - \alpha_{p+1} \, g(\bar{x}_p)}{g(\bar{x}_{p+1}) - g(\bar{x}_p)} \\ \bar{x}_p &= \bar{x}_q + \alpha_p \bar{s}_q \qquad (6) \\ \bar{x}_{p+1} &= \bar{x}_q + \alpha_{p+1} \bar{s}_q \end{aligned}$$

where g ($\bar{X}_p$), g ($\bar{X}_{p+1}$) are constraint function such that g ($\bar{X}_q + \alpha' \bar{S}_q$) $\leq$ e - the accuracy needed.

In using the interior penalty function method, the search is restricted to the feasible region. When $\bar{X}_{p+1}$ violates " g", RANGE progressively reduces the step size to find α′ , which is now expressed as:

$$\alpha' = \alpha_p + \frac{\alpha_{p+1} - \alpha_p}{2} \qquad (7)$$

such that g ($\bar{X}_q + \alpha' \bar{S}_q$) ≤ 0

2.3 ONE-DIMENSIONAL SEARCH METHODS

The range-finder determines the interval which contains the minimum along a search direction. An efficient unidimensional search is next, needed to accurately locate the minimum in a few iterations. The 1-D search problems are of the form:

$$\text{minimize } \emptyset(x)$$

where x is a single variable whose optimum value is to be determined, so that the function Ø is at its minimum.

The program package gives the user, the option of using either the quadratic interpolation scheme (SUBROUTINE QUAD) or the golden section method (SUBROUTINE GOLD) [13]. The former performs a series of iterations approximating Ø(x) as a quadratic function; the latter isolates the minimum in regions of successively decreasing size. The number of iterations needed for convergence to the minimum, will depend on the accuracy needed. Compared to quadratic interpolation scheme, the golden section method converges slowly to the minimum.

2.4 MULTIDIMENSIONAL UNCONSTRAINED SEARCH METHODS

Unconstrained searches, i.e., to just minimize $F(\bar{X})$ without any constraints, can be performed by a sequence of one-dimensional minimizations in appropriate directions. The most efficient search procedures generate directions that are conjugate.

In direct search strategies, the search directions are established apriori but may be modified during the course of a search. In the univariate method (SUBROUTINE UNIV), the design space is searched along the coordinate directions, every time adjusting one variable and keeping others constant. A cycle is completed when all the coordinate directions are searched and repeated, if needed, to satisfy the accuracy criterion. Powell's method (SUBROUTINE CONDIR) begins with the univariate approach but establishes " conjugate" directions by moving along vectors connecting points in the design space, resulting from previous 1-D minimizations. Conjugate directions [1,2,3] lead to speeded convergence of an optimization on most functions and, infact, guarantee convergence on an n-dimensional quadratic function after n, 1-D minimizations in mutually conjugate directions.

Gradient based search techniques utilize gradient information in establishing directions for 1-D minimizations. The objective function has a maximum rate of change along this direction. Thus, moving in the negative gradient direction would seem to reduce the performance index most (SUBROUTINE STEEP). The conjugate gradient (SUBROUTINE CONGRA) and the Davidon-Fletcher-Powell (SUBROUTINE DFPM) methods [1,2,3] utilize the past and current gradient values to generate conjugate directions and speed the convergence. The Davidon-Fletcher-Powell approach is generally considered to be one of the most effective methods for unconstrained optimization. Difficulties seldom arise except on very badly distorted or ecentric functions. When the search direction fails to reduce the function, the search procedure is restarted after adjusting certain parameters accordingly.

2.5 MULTI-DIMENSIONAL CONSTRAINED SEARCH METHODS

The constraints in an optimization problem can be enforced either by using substitutions [1,3] to eliminate them or by modifying the objective function [1,2,3] with penalty terms. The penalty terms increase the objective function when a constraint is violated.

Substitutions and penalty terms transform the constrained problem into a sequence of unconstrained problems. Alternate procedures for obtaining the constrained minimum are based on finding search directions [1,3] that are directed towards the feasible region and away from the constraints just violated. The search for the optimum is confined to the feasible region.

2.5.1 Interior Penalty Function Method

SUBROUTINE IPENAL

The basic idea underlying this method is to solve repetitively a sequence of unconstrained problems whose solutions, in the limit, approach the minimum of the constrained optimization problem. The objective function is modified as

$$\emptyset(\bar{X},R) = F(\bar{X}) - R \sum_{j=1}^{m} \frac{1}{g_j(\bar{X})} + \frac{1}{\sqrt{R}} \sum_{k=1}^{1} [1_k(\bar{X})]^2 \quad \ldots\ldots \quad (8)$$

where Ø is the modified function called PENALTY, R is penalty factor, "m" is the number of inequality constraints, and "1" is the number of equality constraints. The penalty term with inequality constraints is small for points away from the constraints but blows up as the constraints are approached. As R is decreased, the penalty term, with equality constraints 1_k , blows up and it forces 1_k to be satisfied to some degree. When

g_j and l_k are satisfied:

$$\emptyset(\bar{X}^{*},R) \rightarrow F(\bar{X}^{*}) \text{ as } R \rightarrow 0 ,$$

$\bar{X}^{*}$ is the optimum solution.

An extrapolation scheme suggested by Fiacco and McCormick [1,2,3] is used to predict a new starting point and the value of the constrained optimum. The whole search should be strictly within the feasible region, because a mirror image exists in the infeasible region. Thus, precaution is taken in RANGE so as not to overstep into the infeasible region. An appealing feature of this method is that it produces an improving sequence of acceptable designs; hence allowing the designer to compare suboptimal designs to the optimum one.

2.5.2 Exterior Penalty Function Method

SUBROUTINE EXPEN

A frequently used exterior penalty function which accommodates inequality and equality constraints is of the form

$$\emptyset(\bar{X},R) = F(\bar{X}) + R \sum_{j=1}^{m} < g_j(\bar{X}) >^2 + R \sum_{k=1}^{l} [l_k(\bar{X})]^2 \quad .... \quad (9)$$

where $< g_j(\bar{x}) > = 0 \quad \text{for} \quad g_j(\bar{x}) \leq 0$

$\qquad = g_j(\bar{x}) \text{ for } g_j(\bar{x}) > 0$

$\emptyset(\bar{X},R)$ is minimized for increasing values of R. As R is increased, the terms involving g_j and l_k become bigger. The $\bar{X}$ vector is forced by the penalty terms to satisfy the constraints to some degree. As long as g_j and l_k are satisfied, as $\bar{X} \rightarrow \bar{X}^*$, the value of the penalty becomes negligible:

$$\emptyset(\bar{X}^*, R) \longrightarrow F(\bar{X}^*)$$

X^* is the constrained optimum solution. This method converges to the constrained optimum from outside the feasible region for a sequence of unconstrained minimizations with increasing R.

Selection of proper initial values of R depends on the design problem and one's judgement. Fox [1], Rao [3] discussed some rules for picking an initial value of R. A source of trouble in the penalty function methods lies in the relative magnitudes of the constraints. If $g_1 = 1000\ g_2$, g_1 changes more rapidly than g_2, hence, overpowers it over the infeasible region. $\emptyset$ is insensitive to g_2, and the optimum may even violate g_2. This problem is overcome by scaling the constraints.

2.5.3 Feasible Directions Method

SUBROUTINE FEAS

This is based on Zoutendijk's procedure and begins at a point within the constraints, using an unconstrained optimization procedure until a constraint is encountered. Then, a usable feasible direction $\bar{S}$ is defined, which reduces F and does not violate the constraints, by solving the following linear programming problem:

$$\underset{\bar{S},\ \beta}{\text{Max}}\ \beta \ \text{subject to} \quad \left.\begin{array}{l} \bar{S}^T \nabla F + \beta \leq 0 \\ \bar{S}^T \nabla g_j + \theta_j \beta \leq 0 \quad j \in J \end{array}\right\} (10)$$

$$|s_i| \leq 1 \quad i = 1,2,..,n$$

where β is a slack variable, T indicates transpose, J is the set of active constraints and the s_i are the components of $\bar{S}$. The θ_j are the "push-off" factors and, as suggested by Fox, are set to unity for nonlinear constraints and zero for linear constraints.

2.5.4 Convergence Rule

SUBROUTINE CONVRG

For the multi-dimensional search techniques an optional convergence rule is provided. Besides satisfying the accuracy criterion, this option perturbs the design variables and uses a scheme similar to Powell's [1]. It works as follows:

1. Apply the search procedure till the accuracy is achieved. Call the resultant point $\bar{A}$.
2. Perturb the design variables by 1%

3. Apply the normal search until the accuracy is achieved. Call the resultant point $\bar{B}$.

4. Terminate if $F(\bar{A})$ and $F(\bar{B})$ differ by the accuracy desired, or else search for minimum along the line joining $\bar{A}$ and $\bar{B}$ and starting from a point having lower performance index (say $\bar{A}$); call it $\bar{C}$ and neglect $\bar{B}$.

5. Terminate if $F(\bar{C})$ and $F(\bar{A})$ satisfy the accuracy criterion, or else repeat step 1 using $\bar{C}$ as starting point.

The above scheme is less likely to stop prematurely, but it may prove to be time consuming.

Chapters 3 and 4 describe a compact heat exchanger and it's design methodology to help understanding how the optimization package, developed in this chapter, may be used for optimizing the design of a compact heat exchanger.

CHAPTER 3

COMPACT HEAT EXCHANGER

3.1 INTRODUCTION

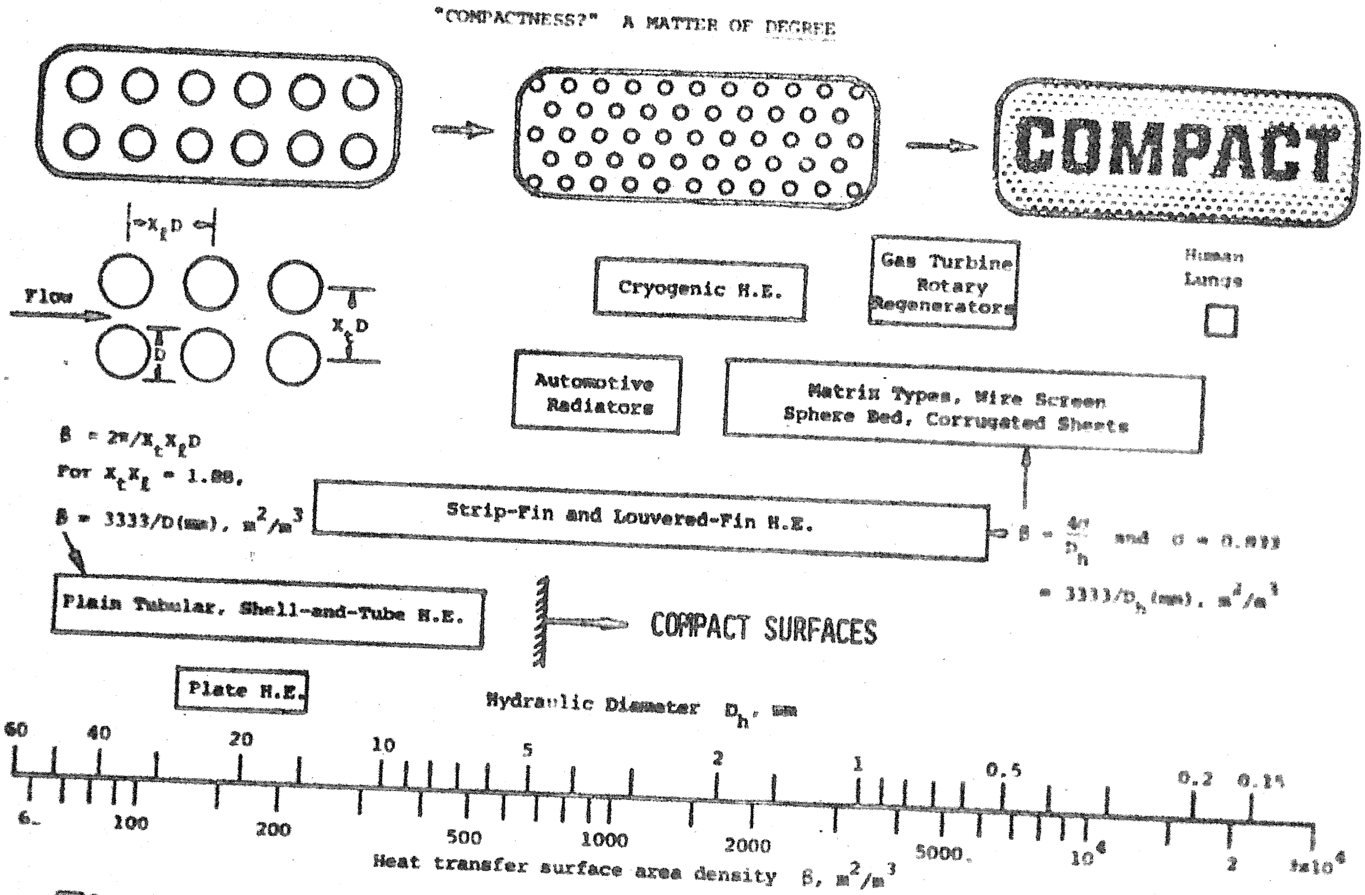
A heat exchanger is a device which provides for transfer of ~~internal~~ thermal energy between two or more fluids at differing temperatures. Heat transfer between fluids takes place through a separating wall. Since the fluids are separated by a heat transfer surface, they do not mix. Common examples of such heat exchangers are the shell-tube-exchangers, automobile radiators, condensers, evaporators, air preheaters. A heat exchanger consists of the active heating elements such as a core or a matrix containing the heat transfer surface, and passive fluid distribution elements such as headers, manifolds, tanks, inlet and outlet nozzles or pipes, or seals. Usually, there are no moving parts in a heat exchanger; however, there are exceptions such as a rotary regenerative exchanger, in which the matrix is mechanically driven to rotate at some design speed.

The heat transfer surface is the surface of the exchanger core which is in direct contact with the fluids and through which heat is transferred by conduction. That portion of the surface which also separates the fluids is referred to as " primary or direct surface". The design thermal effectiveness of exchangers employing such surfaces is usually 60% and below and the heat transfer surface area density is usually less than 300 m^2/m^3. In many applications, a much higher, upto 98% , exchanger effectiveness is essential, and the box volume and mass are limited so that a much more compact surface is mandated. Usually, either a gas or a liquid having a low heat transfer coefficient is the fluid on one or both sides. This results in a large heat transfer surface area requirements. For low density fluids (gases), pressure drop constraints, tend to require a large flow area. So a question arises, how can we increase both the surface area and flow area together in a reasonably shaped configuration. Appendages or fins on the primary surfaces increase the surface area density. Flow area is increased by the use of thin gauge material and sizing the core property. The heat transfer coefficient on the extended surfaces may be higher or lower than that on the unfinned surfaces. For example, the interrupted (strip, louver, etc.) fins provide both an increased area and increased heat transfer coefficient, while the internal fins in a tube, meant primarily for structural strength and flow mixing purposes, may result into a slight reduction in the heat transfer coefficient depending on fin spacing.

Loosely defined,a compact heat exchanger is one which incorporates a heat transfer surface having a high " area density". That is, a high ratio of heat transfer surface to volume. Some what arbitrarily, it is specified that a compact surface has an area density β greater than 700 m^2/m^3. A spectrum of surface area densities of heat exchanger surfaces is shown in Fig. 3. On the bottom of the figure, two scales are shown: the hydraulic diameter D_h in mm and equivalent heat transfer surface area density $\beta(m^2/m^3)$. Different exchanger - surfaces are shown in rectangles. The short vertical sides of a rectangle, when projected on the β (or D_h) scale, indicate the range of surface area density (or hydraulic diameter) for the particular surface in question. Interesting details, for heat exchanger classification have been given [16] with adequate figures. The motivation for using compact surfaces is to gain specified heat exchanger performance, $q/\Delta t_m$, within acceptably low mass and box volume constraints. As

$$\frac{q}{\Delta t_m} = U \beta V ,$$

where q is the heat transfer rate and Δt_m - the mean temperature difference.



"COMPACTNESS?" A MATTER OF DEGREE
COMPACT
$|\Rightarrow X_\ell D \Leftarrow|$
Flow
D
$X_t D$
$\beta = 2\pi/X_t X_\ell D$
For $X_t X_\ell = 1.88$,
$\beta = 3333/D(\text{mm})$, m^2/m^3
Cryogenic H.E.
Gas Turbine Rotary Regenerators
Human Lungs
Automotive Radiators
Matrix Types, Wire Screen Sphere Bed, Corrugated Sheets
Strip-Fin and Louvered-Fin H.E.
$\beta = \frac{4\sigma}{D_h}$ and $\sigma = 0.833$
$= 3333/D_h(\text{mm})$, m^2/m^3
Plain Tubular, Shell-and-Tube H.E.
COMPACT SURFACES
Plate H.E.
Hydraulic Diameter D_h, mm
60
40
20
10
5
2
1
0.5
0.2
0.15
6
100
200
500
1000
2000
5000
10^4
2
3×10^4
Heat transfer surface area density β, m^2/m^3

Fig 3. Heatransfer surface area density spectrum of exchanger surfaces

Clearly, a high β minimizes the exchanger volume V. Moreover, compact surfaces generally result in a higher overall conductance U. This also amounts to having a smaller volume. As compact surfaces can achieve structural stability and strength with a thinner section, the gain in lower exchanger mass is even more pronounced than the gain in lower volume. Various techniques employed to make heat transfer surfaces compact are: fins between plates, finned circular tubes, or densely packed continuous or interrupted cylindrical flow passages of various shapes.

A heat exchanger of any structural construction is considered as compact if it employs a compact surface on either one or more sides of a two-fluid or multi-fluid heat exchanger. The convective heat transfer coefficient for gaseous fluids is generally one or two orders of magnitude lower than those of water, oil, and other fluids. Thus, to reduce the size and weight of a gas-to-liquid heat exchanger, the heat transfer surface on the gas side needs to be much more compact as compared to a simple heat exchanger having only circular tube. Hence, for a somewhat " balanced" design, a compact surface is employed on the gas side. Thus major applications of compact heat exchangers are gas-to-gas, gas-to-liquid, and gas-to-condensing or evaporating fluid heat exchangers.

3.2 CONSTRUCTION TYPES AND SURFACE GEOMETRIES

The basic construction types employed in the design of a compact heat exchanger are extended surface heat exchangers employing fins on one or more sides, regenerators employing small hydraulic diameter surfaces, and tubular exchangers employing small diameter tubes.

Two most common types of extended surface exchangers are the plate-fin and tube-fin types. In a plate-fin exchanger, fins or spacers are sandwiched between parallel plates, referred to as parting plates or parting sheets or formed tubes. Fins are attached to the plates by brazing, soldering, gluing, welding, mechanical fit, or by extrusion. While the plates separate the two fluid streams, the fins form the individual flow passages. Alternate fluid passages are connected in parallel by suitable headers to form two or more sides of the exchanger. The plate-fins are categorized as

1. Plain (uncut surfaces) and straight fins
2. Plain but wavy fins
3. Interrupted fins

Plain triangular and rectangular fins obviously belong to first category. Strip, louver, perforated and pin fins comprise the last category. The heat transfer coefficient and friction factor in the developing flow region are considerably higher than those in the fully developed region. This is because the developing boundary layers are thinner and offer lower thermal and hydro dynamic resistances compared to those for the thick boundary layers associated with the fully developed flows. Wavy and interrupted fins have boundary layers developing after each interruption. With a proper design, the resultant heat transfer coefficients and heat transfer rates are significantly higher at the same pressure drops for wavy and interrupted fins compared to those for the plain fins. Hence, these fins use the materials very efficiently if the design constraints allow the use of such fins.

In a tube-fin exchanger, tubes of round, rectangular or elliptical shapes are generally used. Fins outside the tubes may be categorized as

1. Normal fins on individual tubes
2. Longitudinal fins on individual tubes
3. Continuous (plain, wavy, interrupted) fins on an array of tubes.

Fins inside the tubes are classified as integral or attached fins.

3.3 HEAT EXCHANGER DESIGN METHODOLOGY

Two most common heat exchanger design problems are the rating and sizing problems. For an existing heat exchanger, the performance evaluation problem is referred to as the rating problem. To arrive at a design of a new exchanger to meet the specified performance, is referred to as a sizing problem. The objective in rating is either to verify Vendor's specifications or to determine the performance at off-design conditions. In this problem, the following quantities are specified:the exchanger construction type, flow arrangement, overall core dimensions, complete details on the material and surface geometries on both sides including their heat transfer and pressure drop characteristics, fluid flow rates, inlet temperatures, and fouling factors. The designer's task is to predict the fluid outlet temperatures, heat transfer rate, and pressure drop on each side.

Sizing involves a design of a new heat exchanger for specified performance within known constraints and minimum objective function. Here the fluid inlet and outlet temperatures and flow rates are generally specified as well as the pressure drop on each side. The designer selects the construction type, flow arrangement, materials and surfaces on each side to determine the core dimensions to meet the specified heat transfer and pressure drop requirements. If the problem is not well posed without even minimum necessary information, the designer arrives at the necessary information based on the discussions with the customer, his own experience and engineering judgements.

The package approach suggested by Shah et al. [12] has been adopted in the present work for the heat exchanger design. The complete procedure is given in the flow chart form (Fig. 4.) In this method each possible surface geometry and construction type is considered to be an alternative design. Thus there may be several independent optimized solutions satisfying the problem requirements. Engineering judgement, comparison of objective function values, and other evaluation criteria are then applied to select a final optimum solution for implementation. Thus after ariving at the possible alternate designs, the total number of constraints for each design are formulated. This includes the customer's explicitly specified constraints like fixed frontal area, ranges of heat exchanger dimensions and implicit constraints like minimum heat transfer, and allowable pressure drop. Starting with a set of dimensions for the geometry and input operating conditions, the rating problem is solved. The output from the heat exchanger calculations is fed to the optimization package where the constraints and objective function are evaluated. New values for the design variables are subsequently generated and heat exchanger calculations are repeated. The iterations are continued until the objective function is

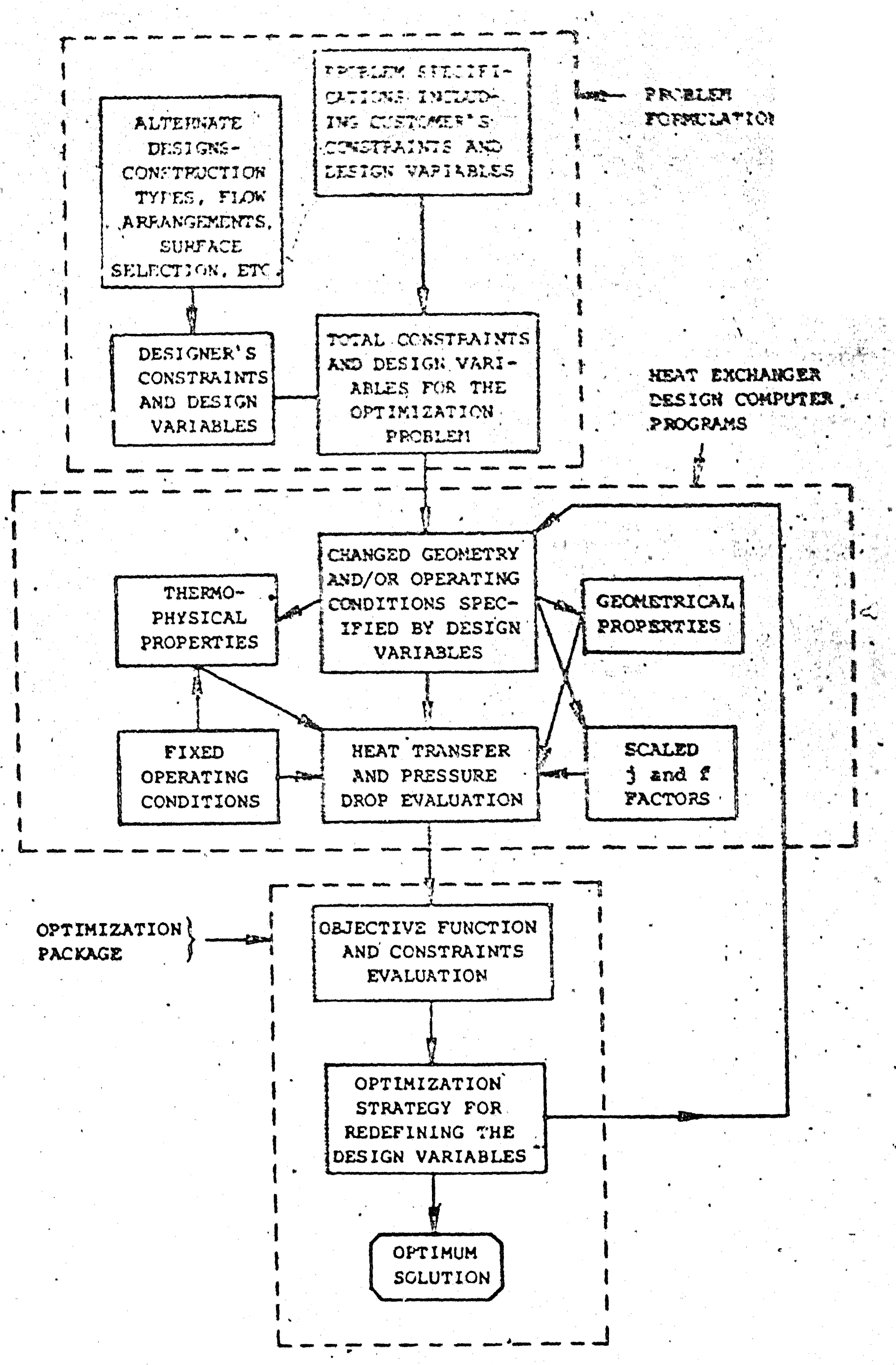

PROBLEM SPECIFICATIONS INCLUDING CUSTOMER'S CONSTRAINTS AND DESIGN VARIABLES
PROBLEM FORMULATION
ALTERNATE DESIGNS-CONSTRUCTION TYPES, FLOW ARRANGEMENTS, SURFACE SELECTION, ETC.
DESIGNER'S CONSTRAINTS AND DESIGN VARIABLES
TOTAL CONSTRAINTS AND DESIGN VARIABLES FOR THE OPTIMIZATION PROBLEM
HEAT EXCHANGER DESIGN COMPUTER PROGRAMS
CHANGED GEOMETRY AND/OR OPERATING CONDITIONS SPECIFIED BY DESIGN VARIABLES
THERMO-PHYSICAL PROPERTIES
GEOMETRICAL PROPERTIES
FIXED OPERATING CONDITIONS
HEAT TRANSFER AND PRESSURE DROP EVALUATION
SCALED j and f FACTORS
OPTIMIZATION PACKAGE
OBJECTIVE FUNCTION AND CONSTRAINTS EVALUATION
OPTIMIZATION STRATEGY FOR REDEFINING THE DESIGN VARIABLES
OPTIMUM SOLUTION

Fig 4· Heat exchanger optimization methodology optimized within the desired accuracy and all the constraints are satisfied.

A glance at ~~the~~ Fig. 4 indicates two computer program packages needed for the optimization are the heat exchanger design programs and the optimization package. It should be emphasized that the heat exchanger programs must be working properly before any attempt is made to implement the optimization package. Otherwise, "garbage-in gospel-out" will result from the "black box".

CHAPTER 4

OPTIMUM DESIGN OF A COMPACT HEAT-EXCHANGER

4.1 INTRODUCTION

A variety of heat-exchanger design-methods have been proposed to determine the best performance of a heat exchanger based on specific performance criteria. However, little work [12, 14] has been done applying the concepts and techniques of non-linear programming to optimizinge heat exchanger design. These techniques offer a systematic approach for improving heat exchanger performance by allowing trade-offs to be made between competing objectives such as pressure drop, power, cost, volume and weight. The optimization package developed in Chapter-2 is quite general and may be used for any design problem. The present chapter is devoted to test the applicability and efficacy of the general package for the design of a compact heat exchanger for a gas-turbine plant. Relevant datas for the design parameters have been taken from Appendix-B of Kays and London [15]. The objective is to minimize pumping power requirements with constraints on the exchanger size, volume and the total heat transferred. The design variables are the hot and cold fluid capacity flows and linear dimensions of the heat exchanger. The ε - NTU approach is used to evaluate the heat exchanger performance. The design procedure outlined in this chapter is quite general and suitable for any heat exchanger application.

4.2 NOMENCLATURE

A_a	Exchanger total heat transfer area on air side
A_c	Free flow area
A_f	Exchanger total fin area on air side
A_{fra}	Exchanger total frontal area on air side
A_{frw}	Exchanger total frontal area on water side
B	Exchanger breadth (Fig. 5)
C_a	Flow-stream capacity rate of air
C_w	Flow-stream capacity rate of water
C_{min}	Minimum of C_a or C_w
C_{max}	Maximum of C_a or C_w
C_{pa}	Specific heat at constant pressure for air
C_{pw}	Specific heat at constant pressure for water
f_a	Friction factor on air side
f_w	Friction factor on water side
G_a	Air stream mass velocity
G_w	Water stream mass velocity
g_c	Proportionality factor in Newton's second law
h_a	Heat transfer film coefficient on air side

h_w	Heat transfer film coefficient on water side
H	Exchanger height (Fig. 5)
k_a	Thermal conductivity of air
k_w	Thermal conductivity of water
k_f	Thermal conductivity of fin material
L	Exchanger fluid-flow length
l_f	Fin length
M	Molecular weight of air
m	A fin effectiveness parameter
P_a	Pressure on air side
P_w	Pressure on water side
Q	Total heat transferred
Q_D	Total desired heat transfer
R	Universal gas constant
$C*$	Capacity ratio C_{min}/C_{max}
r_a	Hydraulic radius on air side
r_w	Hydraulic radius on water side
T_c	Temperature of cold fluid (water)
T_h	Temperature of hot fluid (air)
U	Overall thermal conductance
V	Volume of heat exchanger
v	Specific volume of air
W	Exchanger width (Fig. 5)

Greek Symbols

α	Ratio of total transfer area on one side of the exchanger to total volume of the exchanger
δ_f	Fin thickness
ε	Exchanger effectiveness
η_f	Fin temperature effectiveness
η	Total surface temperature effectiveness
σ	Ratio of free flow area to frontal area
μ	Viscosity coefficient
ρ	Density

Dimensionless Groupings

Re	Reynolds number $(4\ r\ G/\mu)$
Pr	Prandtl number $(\mu\ C_p/k)$
NTU	Number of transfer units (AU/C_{min})
j	Colburn factor $(h\ P_r^{2/3}\ /\ G\ C_p)$

Subscripts

a	Air side
avg	Average
i	Inlet conditions
o	Outlet conditions

4.3 PROBLEM FORMULATION

4.3.1 Design Problem

The heat exchanger configuration is shown in Fig.5. The inlet pressure, temperature and flow stream capacity rate of the hot air are T_{hi} (126.7°C), P_a (275.8 kP_a) and C_a respectively. The values in the parenthesis have been chosen for a specific application (A GAS TURBINE HEAT EXCHANGER). The hot air is cooled by water, pumped at the rate of C_w and at ambient temperature of T_{ci} (15.6°C) and pressure P_w (101.04 kP_a). It is required to minimize the total fluid pumping power for this exchanger by varying the heat capacity rates C_a , C_w on each side and also varying the height, width and breadth of the exchanger. It should be spaced within a volume of V_{max} and it has a heat transfer rate of atleast Q_D.

4.3.2 Selection Aspects

First of all, a cross flow arrangement with both fluids unmixed is selected. For a complete optimization, every possible combination of available heat transfer surfaces should be considered in order to make, reasonably, sure, that the optimum design is obtained. This may be impractical and ~~too~~ time consuming. For the given pressure, temperature ranges and space constraint, as selected for the present work in 4.3.1, a finned-flat tube core configuration is suitable [15]. Once the basic surface geometry

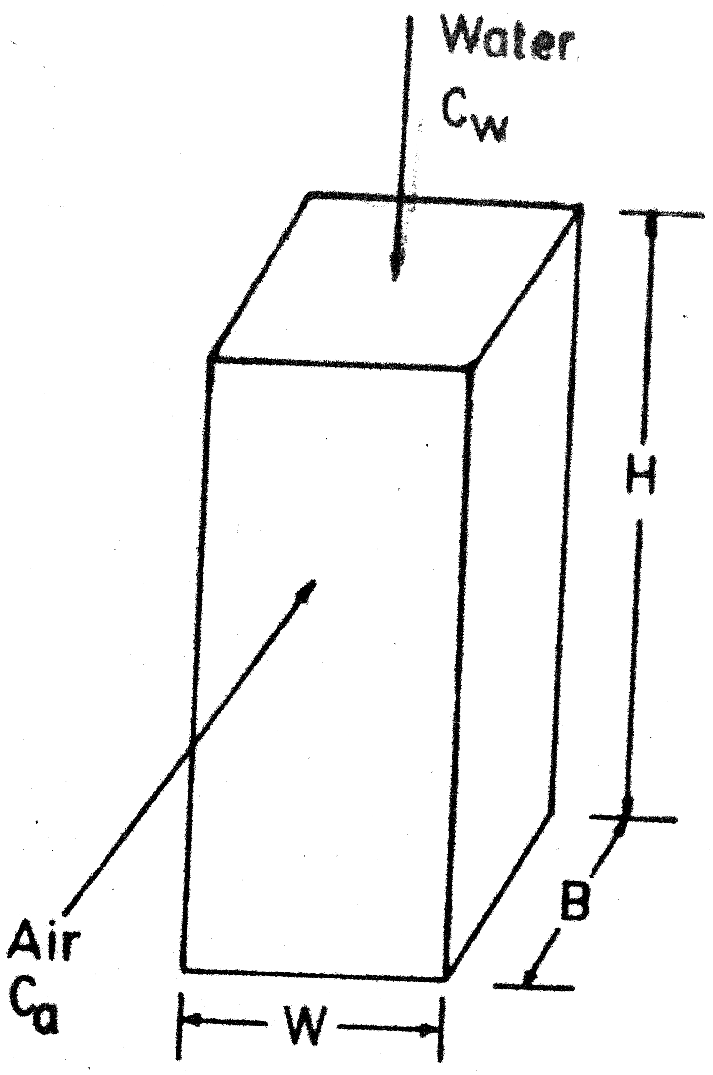


Fig 5. Heat exchanger Configuration & Design variables

is selected, the designer gets the values of various geometric parameters, as listed below, that enter the heat transfer calculation.

1. Hydraulic radius, r
2. Heat transfer area/Total volume, α
3. Free flow area/Frontal area, σ
4. Fin area/Heat transfer area, A_f/A
5. Fin thickness, δ_f
6. Fin conductivity, k_f
7. Fin length, l_f.

These are to be determined for the hot and cold fluid sides of the heat exchanger. The heat core selected in this application uses fins only on the air side for the obvious reason that the heat transfer film coefficient is lower on this side. The friction and colburn factors for the flow inside the flattened tubes are obtained from [15] assuming the ratio $L/4r_h = 40$, and are given in figures 6 and 7.

4.3.3 Optimization Problem Definition

The design variables are exchanger width W, height H, breadth B, and the capacities C_a and C_w. By appropriately scaling and nondimensionalizing these variables, a new set of parameters may be defined which have the same order of magnitude. This makes the operation of optimization procedure more convenient. Defining

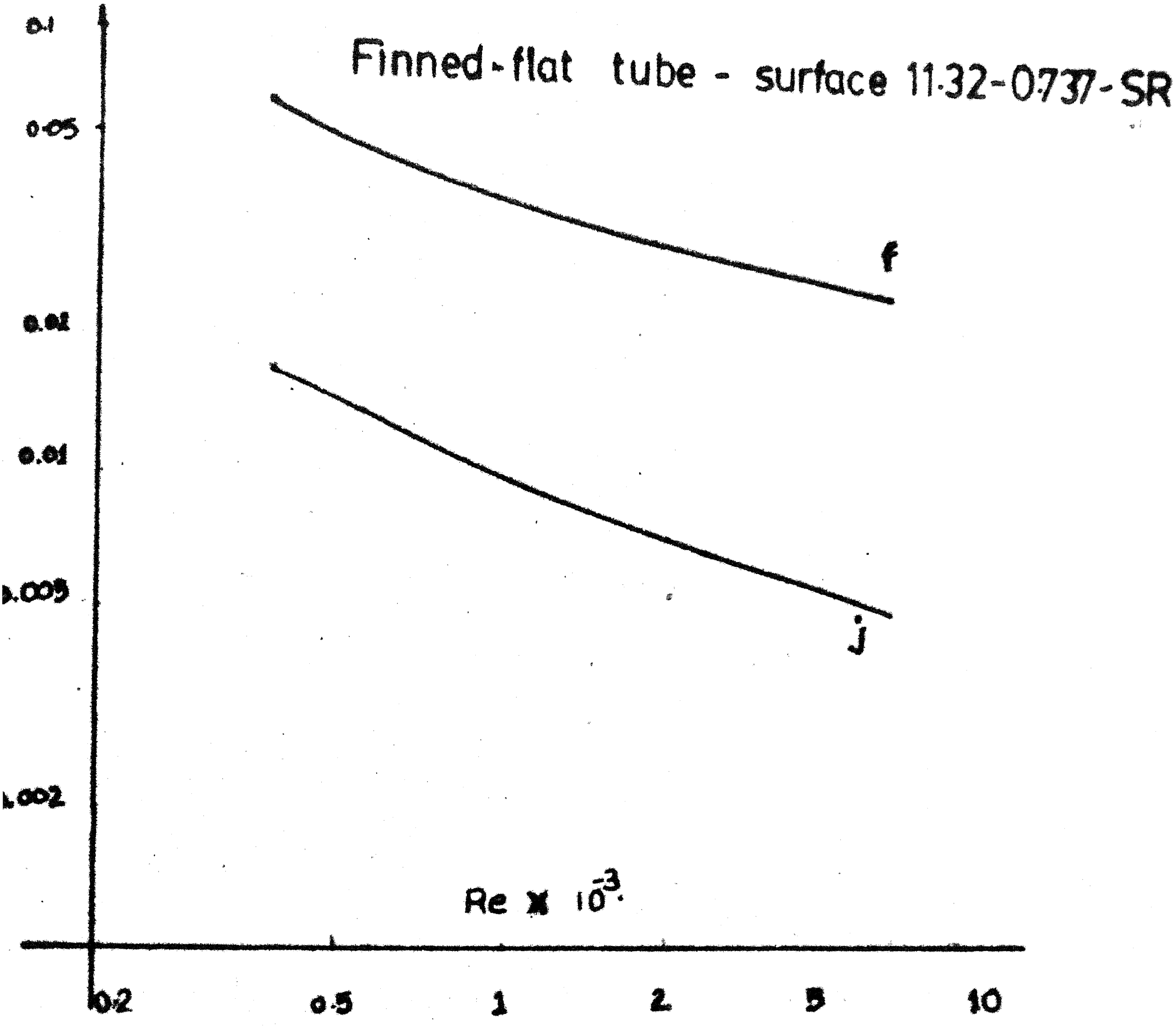


Fig 6. Friction & Colburn factors forAir side

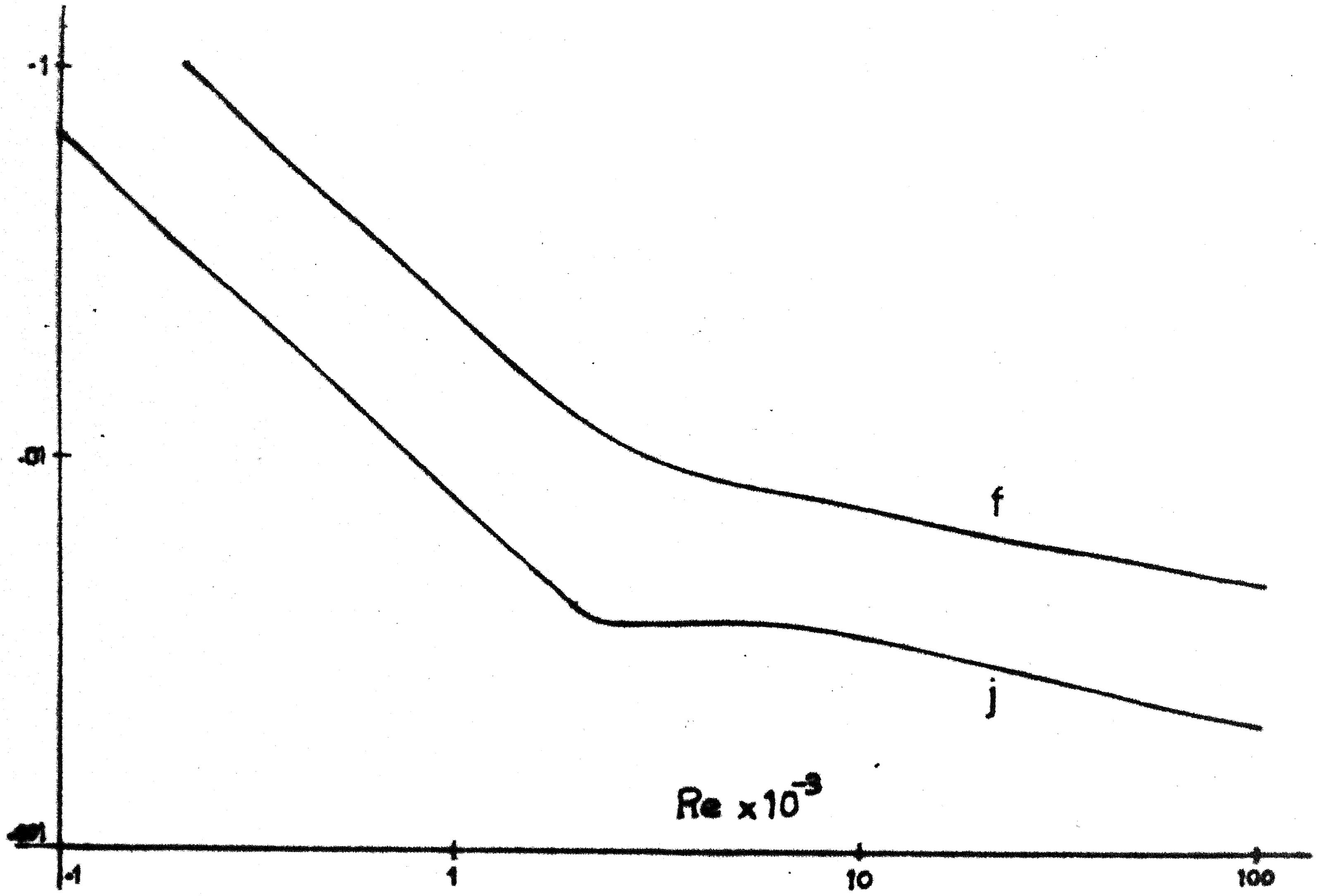


Fig 7. Friction & Colburn factors for Tube side

$$L = (V_{max})^{1/3} \qquad (11)$$

$$Z = \frac{Q_D}{T_{hi} - T_{ci}}$$

the variables W, H, B, C_a and C_w are nondimensionalized and the design vector is defined as follows:

$$X_1 = C_a/Z$$

$$X_2 = C_w/10Z \qquad (12)$$

$$X_3 = W/L$$

$$X_4 = B/L$$

$$X_5 = H/L$$

Representing the pumping power as Ø, the minimization problem is:

$$\text{Minimize} \quad \emptyset\ (\bar{X}) \qquad (13)$$

Subject to the following constraints.

i) To maintain the minimum value of the heat transfer rate, $Q \geq Q_D$

$$\therefore \quad g_1 = 1 - \frac{Q}{Q_D} \leq 0$$

ii) To space the exchanger within a certain volume $V \leq V_{max}$

$$\therefore \quad g_2 = \frac{V}{V_{max}} - 1 \leq 0$$

iii) Realizing the need for a meaningful configuration, the following constraints are imposed:

Width should be less than twice the breadth, and Height should be less than thrice the width i.e.

$$X_3 \leq 2\ X_4$$

$$X_5 \leq 3\ X_3$$

$$g_3 = \frac{X_3}{2X_4} - 1 \leq 0$$

$$g_4 = \frac{X_5}{3X_3} - 1 \leq 0$$

iv) Upper bounds for the capacities are given some practical values say BIG

$$C_a < \text{BIG}$$

$$C_w < 10\ \text{BIG}$$

The water capacity rate is taken to be of one order higher than air capacity rate.

$$g_5 = \frac{X_1}{\text{BIG}} - 1 \leq 0$$

$$g_6 = \frac{X_2}{10\ \text{BIG}} - 1 \leq 0$$

v) Upper bound for the remaining variable breadth is given as 1/L so as to satisfy the volume constraint always:

$$g_7 = L\ X_4 - 1 \leq 0$$

vi) Non-negativity constraints are finally imposed on all the five variables.

$$g_8 = - X_1 < 0$$

$$g_9 = - X_2 < 0$$

$$g_{10} = - X_3 < 0$$

$$g_{11} = - X_4 < 0$$

$$g_{12} = - X_5 < 0$$

4.4 PERFORMANCE CALCULATION PROCEDURE

4.4.1 Overview

The heat exchanger performance is evaluated in an iterative manner as shown in Fig. 8. An iterative scheme has to be used since the outlet fluid temperatures and heat exchanger effectiveness are not known at the outset. A value of 0.75 is assumed for the effectiveness ε at the start of the iteration scheme. The outlet fluid temperatures can then be found. The fluid properties vary with temperature and are evaluated at the mean of the inlet and outlet temperatures. The fluid properties appear in the expressions for mass velocities (G), and Reynolds (Re) and Prandtl (Pr) numbers. The Colburn 'j' and friction 'f' factors are function of Re. With j and Pr known, the film coefficient h can be computed, then the overall conductance U, and finally the number of transfer units NTU. Exchanger effectiveness ε is a

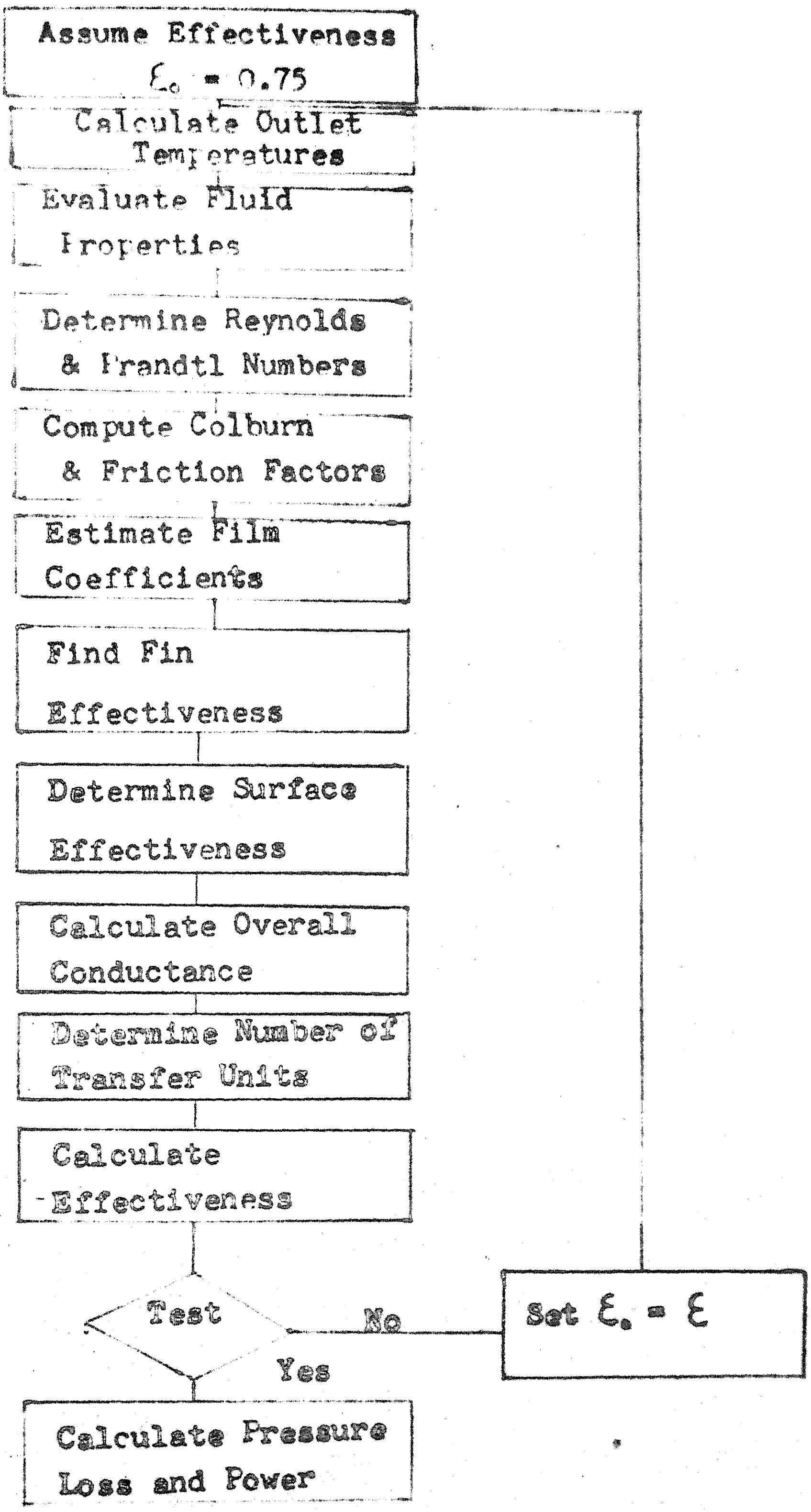

Assume Effectiveness
$\mathcal{E}_o = 0.75$
Calculate Outlet Temperatures
Evaluate Fluid Properties
Determine Reynolds & Prandtl Numbers
Compute Colburn & Friction Factors
Estimate Film Coefficients
Find Fin Effectiveness
Determine Surface Effectiveness
Calculate Overall Conductance
Determine Number of Transfer Units
Calculate Effectiveness
Test
No
Yes
Set $\mathcal{E}_o = \mathcal{E}$
Calculate Pressure Loss and Power

Fig 8 . PERFORMANCE ANALYSIS CHART

function of NTU and capacity ratio C^{*}. Expressions for ε are available for different flow configurations. The computed value of ε is then compared with the assumed value for concurrency.

4.4.2 Core Dimensions

The geometrical parameters for the selected finned-flat tube surface 11.32-0.737-SR [15] are:

i) Air side:

$$\begin{aligned} r_a &= 0.00288 \\ \alpha_a &= 270 \\ A_f/A_a &= 0.845 \\ \sigma_a &= 0.78 \\ \delta_f &= 0.00033 \\ k_f &= 100 \\ l_f &= 0.01875 \end{aligned} \tag{14}$$

ii) Water side:

$$\begin{aligned} r_w &= 3.06 \times 10^{-3} \\ \alpha_w &= 42.1 \\ \sigma_w &= 0.129 \end{aligned} \tag{15}$$

From Fig. 5, the frontal areas for the air and water,sides of the intercooler can be readily computed. The frontal area is the cross-sectional area normal to the direction of the flow.

$$A_{fra} = WH$$

$$= X_3\ X_5\ L^2$$

$$A_{frw} = WB \qquad (16)$$

$$= X_3\ X_4\ L^2$$

The total volume of the heat exchanger is :

$$V = WHB$$

$$= X_3\ X_4\ X_5\ L^3 \qquad (17)$$

4.4.3 Outlet Fluid Temperatures

From equation(12) we have

$$C_a = X_1\ Z$$

$$C_w = 10\ X_2 Z \qquad (18)$$

Defining C_{min} to be the minimum of C_a and C_w , the maximum possible amount of heat that can be transferred by the heat exchanger is given by:

$$Q_{max} = C_{min}\ (T_{hi} - T_{ci}) \qquad (19)$$

If the intercooler effectiveness is ε, then the actual amount of heat exchanged between the hot and cold fluid streams is:

$$Q = \varepsilon\ Q_{max}$$

$$= \varepsilon\ C_{min}\ (T_{hi} - T_{ci}) \qquad (20)$$

The heat balance equations are:

$$\begin{aligned} Q &= C_a \, (T_{hi} - T_{ho}) \\ &= C_w \, (T_{co} - T_{ci}) \end{aligned} \tag{21}$$

Substituting (20) in (21), the outlet temperatures are:

$$\begin{aligned} T_{ho} &= T_{hi} - \frac{\varepsilon \, C_{min} \, (T_{hi} - T_{ci})}{C_a} \\ T_{co} &= T_{ci} + \frac{\varepsilon \, C_{min} \, (T_{hi} - T_{ci})}{C_w} \end{aligned} \tag{22}$$

The optimizer will manipulate the design variables in order to find optimum, ensuring at the same time that the above energy balance is satisfied.

4.4.4 Fluid Properties

For heat exchanger applications, it is a normal practice to estimate the transport properties, which are temperature dependent, at the bulk temperature which is the average of the fluid inlet and outlet temperatures. The bulk temperatures for the air and water streams are:

$$\begin{aligned} T_{cavg} &= \frac{T_{co} + T_{ci}}{2} \\ T_{havg} &= \frac{T_{ho} + T_{hi}}{2} \end{aligned} \tag{23}$$

Experimental data for the fluid properties of air and water at different temperatures are readily available[19].

In this optimization study, polynomial expressions are fitted to the experimental data using the least squares technique (Fig. 9). The perfect gas equation, Pv=RT/M, is used to determine the specific volume of air at the inlet, outlet, and bulk temperatures with the pressures assumed to remain constant at P_a.

$$v_i = \frac{53.345\ (460 + T_{hi})}{133\ P_a}$$

$$v_o = \frac{53.345\ (460 + T_{ho})}{144\ P_a} \qquad (24)$$

$$v_{avg} = \frac{53.345\ (460 + T_{havg})}{144\ P_a}$$

4.4.5 Reynolds and Prandtl Number

The mass flow is given by dividing the heat capacity C by the specific heat C_p. With the free flow area known, the mass velocities for air and water are:

$$G_a = \frac{C_a}{C_{pa}\ A_{ca}} = \frac{C_a}{C_{pa}\ \sigma_a\ A_{fra}}$$

$$G_w = \frac{C_w}{C_{pw}\ A_{cw}} = \frac{C_w}{C_{pw}\ \sigma_w\ A_{frw}} \qquad (25)$$

The Reynolds and Prandtl numbers can now be computed as:

$$Re_a = \frac{4 \cdot G_a \cdot r_a}{\mu_a}$$

$$Re_w = \frac{4 \cdot G_w \cdot r_w}{\mu_w} \qquad (26)$$

$$Pr_a = \frac{\mu_a \ C_{pa}}{k_a}$$

$$Pr_w = \frac{\mu_w \ C_{pw}}{k_w}$$

4.4.6 Friction and Colburn Factors

Polynomial expressions, Fig. 9 are fitted to the experimental data as shown in Figures 6 and 7 by least square method. In practice, the experimental data is approximated by several polynomials which are valid for a small range of Reynolds number. Since most compact heat exchangers operate in the transition flow regime ($100 < Re < 10,000$) it is important to get good polynomial fits for f and j in this flow regime.

4.4.7 Heat Transfer Film Coefficient

Once the colburn factors for the hot and cold fluids are known, the film coefficients are calculated from the following equations:

$$h_a = \frac{j_a \ G_a \ C_{pa}}{(Pr_a)^{2/3}}$$

$$h_w = \frac{j_w \ G_w \ C_{pw}}{(Pr_w)^{2/3}} \qquad (27)$$

Expression for Fluid Properties :

Water

$$\text{Let } G = \frac{T_{cavg}}{100}$$

$$C_{pw} = .00178\,G^4 - .01789\,G^3 + .06763\,G^2 - .09169\,G + 1.0348$$

$$\rho_w = 10\,(-.00193\,G^4 + .02180\,G^3 - .1300\,G^2 + .08017\,G + 6.2283\,)$$

$$\mu_w = .0547\,G^4 - .56414\,G^3 + 2.6107\,G^2 - 3.5424\,G + 5.1444$$

$$k_w = .1\,(-.00332\,G^4 + .04371\,G^3 - .295151\,G^2 + .88891\,G + 3.0051\,)$$

Air

$$\text{Let } G = \frac{T_{havg}}{1000}$$

$$C_{pa} = .1\,(.03307\,G^4 - .19583\,G^3 + .30682\,G^2 + .20886\,G + 2.3653\,)$$

$$\mu_a = .1\,(-.00571\,G^4 + .05756\,G3 - .22801\,G^2 + .67541\,G + .40343\,)$$

$$k_a = .01\,(.00735\,G^4 - .04446\,G^3 - .10931\,G^2 + 2.0579\,G + 1.4882\,)$$

Friction and Colburn Factors :

Valid for 1000 **Re** 10,000

Air

$$\text{Let } Y = \log_{10}(\,Re_a * 10^{-3})$$

$$\log_{10}(100f_a) = .11998Y^4 - .04463\,Y^3 + .26677\,Y^2 - .45965\,Y + .58784$$

$$\log_{10}(100j_a) = -.11873\,Y^4 + .03586\,Y^3 + .22707\,Y^2 - .4789\,Y - .00622$$

Water

$$\text{Let } Y = \log_{10}(\,Re_w * 10^{-3}\,)$$

$$\log_{10}\,(100f_w) = -1.2239\,Y^4 + 2.1643\,Y^3 - .56969\,Y^2 - .87268\,Y + .38629$$

$$\log_{10}\,(100j_w) = -1.3431\,Y^4 + 1.6032\,Y^3 + .5549\,Y^2 - 1.2064\,Y - .08379$$

ig 9 . EXPRESSIONS FOR FLUID PROPERTIES & f and j

4.4.8 Fin Effectiveness

For finned-tube compact surfaces, the fins are assumed to be straight uniform fins. The fin effectiveness for this type of fin is shown to be [20].

$$\eta_f = \frac{\tanh (ml_f)}{ml_f} \tag{28}$$

where

$$m = \left(\frac{2h_a}{k_f \delta_f}\right)^{1/2}$$

The surface effectiveness of the heat transfer surface on the air side is given by [20]:

$$\eta = 1 - \frac{A_f}{A_a} (1 - \eta_f) \tag{29}$$

4.4.9 Exchanger Effectiveness

The conductance of the heat transfer surface can be computed from the film coefficients and the surface effectiveness. In computing the overall conductance, the wall resistance of the surface is neglected.

$$\frac{1}{U_a} = \frac{1}{\eta h_a} + \frac{1}{\frac{\alpha_w}{\alpha_a} h_w} \tag{30}$$

Equation (30) gives the overall conductance based on the air side heat transfer area. The number of transfer units (NTU) is

$$\mathrm{NTU} = \frac{A_a U_a}{C_{min}} = \frac{\alpha_a V U_a}{C_{min}} \tag{31}$$

Let C_{max} represent the maximum of C_a and C_w. Then the capacity ratio C^* is:

$$C^* = \frac{C_{min}}{C_{max}} \qquad (32)$$

With NTU and C^* known, the effectiveness ε for the crossflow intercooler with both fluids unmixed, is determined from the series solution of Mason [21]

$$\varepsilon = \frac{1}{(NTU)C^*} \sum_{n=o}^{\infty} \left[1-e^{-NTU} \sum_{m=o}^{n} \frac{(NTU)^m}{m} \right.$$
$$\left. 1-e^{-(NTU)C^*} \sum_{m=o}^{n} \frac{[(NTU)C^*]^m}{m} \right] \qquad (33)$$

This series is rapidly convergent. The value of ε from (33) is compared with the value assumed at the beginning of the iteration. If these two values differ by more than 0.01, the performance calculations are repeated, using the new value ε for the next iteration. A few iterations are needed to obtain agreement between the assumed and computed effectiveness of the compact heat exchanger.

4.4.10 Pressure Drops

The pressure loss in the fluid streams is contributed by the change in fluid momentum, resulting from the changes in the cross-sectional area at inlet and exit, and by viscous friction. The pressure loss for the air stream is given by [15]

$$\frac{\Delta p_a}{P_a} = \frac{\left(\frac{G_a}{3600}\right)^2 v_i}{2g_c\, P_a} \left[(1+\sigma_a^{\,2})\left(\frac{v_o}{v_i}-1\right) + f_a\,\frac{A_a}{A_c}\,\frac{v_{avg}}{v_i}\right]$$

$$\ldots \qquad (34)$$

From the definition of hydraulic diameter:

$$\frac{A_a}{A_c} = \frac{B}{r_a} = \frac{X_4 L}{r_a} \qquad (35)$$

On substituting (35) in (34), we get

$$\frac{\Delta P_a}{P_a} = \frac{\left(\frac{G_a}{3600}\right)^2 v_i}{2g_c\quad P_a} \left[(1+\sigma_a^{\,2})\left(\frac{v_o}{v_i}-1\right) + \frac{f_a\,X_4 L}{r_a}\,\frac{v_{avg}}{v_i}\right]$$

$$\ldots \qquad (36)$$

The pressure loss associated with the water stream is mainly due to surface friction and is given by:

$$\begin{aligned} \Delta P_w &= \left(\frac{G_w}{3600}\right)^2 \frac{f_w\,H}{2g_c\,\rho\,r_w} \\ &= \left(\frac{G_w}{3600}\right)^2 \frac{f_w\,X_5 L}{2g_c\,\rho\,r_w} \end{aligned} \qquad (37)$$

where, g_c is the gravitational constant, and ρ is the average mass density of water.

The pumping power for each media is the product of pressure loss and volume flow rate, and the total power is given by:

$$\emptyset = \Delta P_a \cdot \frac{C_a \cdot \dot{V}_{avg}}{C_{pa}} + \frac{\Delta P_w\; C_w}{\rho\;\, C_{pw}} \qquad (38)$$

4.5 SOLUTION TECHNIQUES

Three different constrained minimization techniques were applied to minimize the pumping power (Eqn. 13). They are Interior penalty function, Exterior penalty function and the Method of feasible directions. For penalty function methods, the multidimensional minimization schemes tried were Steepest descent, Powell's conjugate directions, Fletcher and Reeves' conjugate gradient and the Davidon-Fletcher-Powell variable metric method. For obvious reasons, Univariate method was ignored. These methods were made to select the Quadratic interpolation procedure for the unidimensional minimization. A graphics program, using the GPGS (General Purpose Graphics System) routines, displays on the graphics terminal the heat exchanger configuration according to the optimized dimensions. This may be used also to give the initial solution for linear dimensions of the exchanger, at the start of the execution of the optimizer, to suit the designer's taste for configuration.

The heat exchanger performance analysis is put in a separate routine, Appendix-A. Finite difference schemes were utilized to get the gradients for various search techniques in the optimizer. All the input parameters used for the heat exchanger performance analysis have been defined in sections 4.2 and 4.3.

A sample of the input data, for the optimizer, used to define the variables and initialize the flags is given here. Results are given in the graphical form. Variation of pumping power and effectiveness with the volume, has been plotted in figures 10 and 11,and a detailed discussion is reserved for the next chapter.

List of Input Data			Significance
NN	=	5	Number of design variables
NC	=	12	Total number of constraints
NI	=	12	Number of inequality constraints
X(1)	=	2.5	Starting point (Feasibility is a must for Interior penalty and Feasible direction methods)
X(2)	=	1.0	
X(3)	=	0.5	
X(4)	=	0.3	
X(5)	=	1.25	
VMAX	=	16.0	Max. volume allowed in a specific run
BIG	=	5.0	Max. capacity rate of both fluids
TT	=	0.1	Initial step size
TMAX	=	0.5	Max. step size allowed
LIM	=	15	Max. number of iterations permitted
FF	=	0.01	Percentage change in variables for FDM computation of gradients
ACC	=	1.E-04	Accuracy desired
R	=	10	Initial, penalty factor

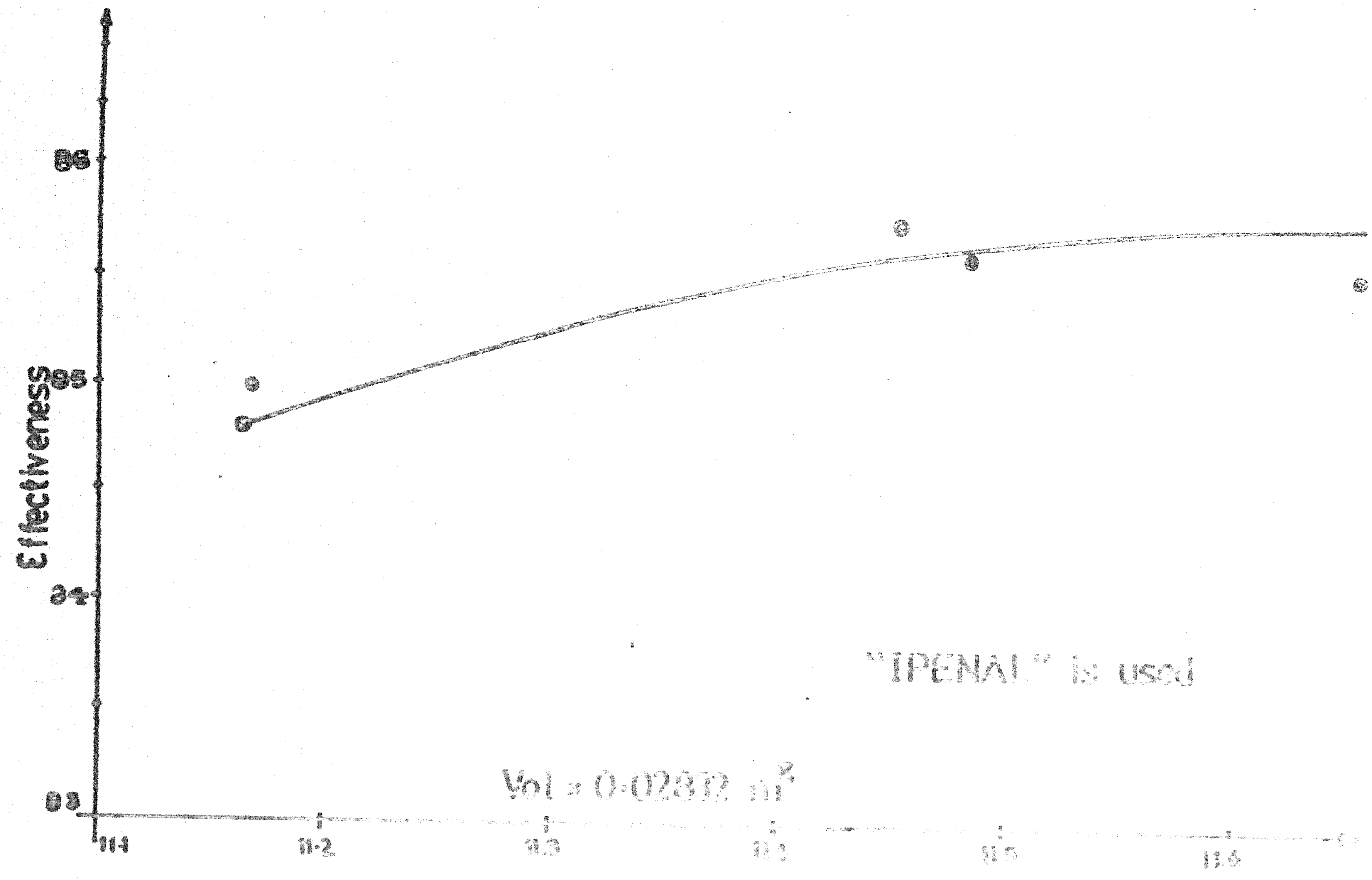


Fig 10. Variation of Effectiveness with Vol_{opt}

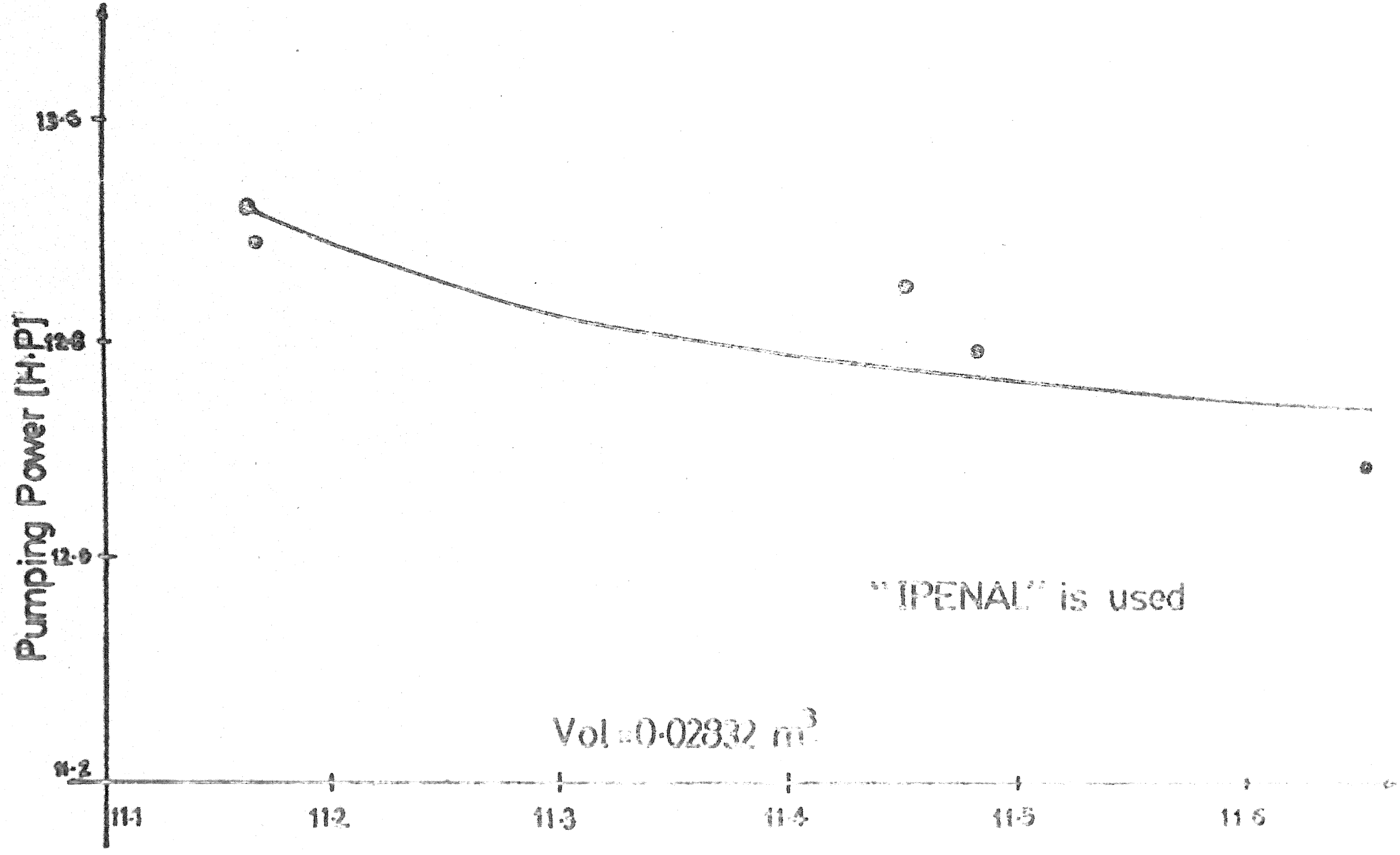


Fig 11. $\mathcal{P}$ vs Vol$_{opt}$

FC	=	0.1	Factor decreasing R for IPENAL
	=	10	Factor increasing R for EXPEN
W(1:NC)	=	0	Push off factors set to ‘O’ for linear constraints
NPERT	=	1	Skips the perturbation scheme

CHAPTER - 5

RESULTS AND DISCUSSION

5.1 TRADE-OFF STUDIES

The compact heat exchanger optimized in the last chapter has the same configuration as considered by Kays and Londan in example-1 [15]. The objective of the study is to find what reductions in pumping power or volume is possible for this design, while still keeping the same heat transfer performance. The heat exchanger is designed by varying it's dimensions and the mass flow rates of the two fluids viz, air and water. The results, as given in figures 10 and 11, are indicative of the characteristics of a single optimization. Fig. 10 indicates the variation of pumping power with the exchanger volume and Fig. 11 is drawn for effectiveness of the exchanger versus volume. Each point on the curve in Fig. 10 corresponds to the optimum value of the pumping power for a specific size of the exchanger. Similarly, in Fig. 11, the optimum performance of the exchanger is indicated for a particular volume.

However, an expanded optimization study can easily be done by changing the appropriate constraints.

For example, if one is interested in finding the fluid pumping power requirement as a function of the exchanger total volume, V_{max} is varied and the resultant optimization problem is solved again by the methods used in the last chapter. Optimization results of eqns.(13) for different values of V_{max} showed that the constraints on linear dimensions of the exchanger are always satisfied and hence, x_3 , x_4 and x_5 are determined by solving these constraint equation as:

$$x_3 = 0.8736$$

$$x_4 = 0.4368$$

$$x_5 = 2.6207$$

The minimization problem in (13) is now reduced to a 2 variables problem and can be restated as

$$\underset{\bar{X}}{\text{Minimize}} \qquad \emptyset\ (x_1\ ,\ x_2)$$

subject to

$$g_1\ (x_1\ ,\ x_2) = 1 - \frac{Q\ (x_1\ ,\ x_2)}{Q_D}$$

This two-variables problem is solved by the same three different methods of constrained minimization for different values of V_{max}. A trade-off curve between pumping power and exchanger volume is presented in Fig. 12, variation of effectiveness with volume in Fig. 13, and variation of mass flow rates with the volume is shown in Fig. 14.

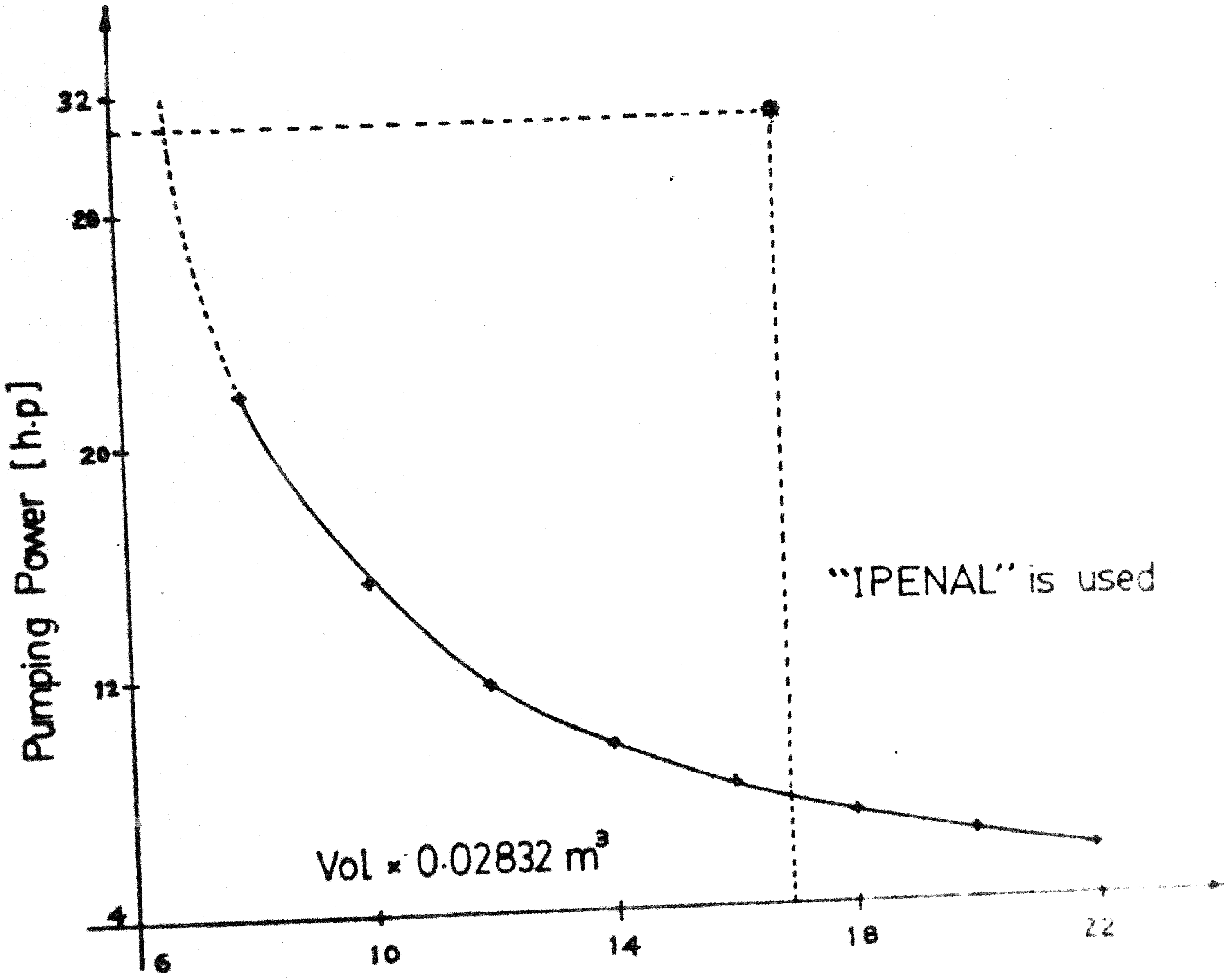


Fig 12. Trade-off- Curve

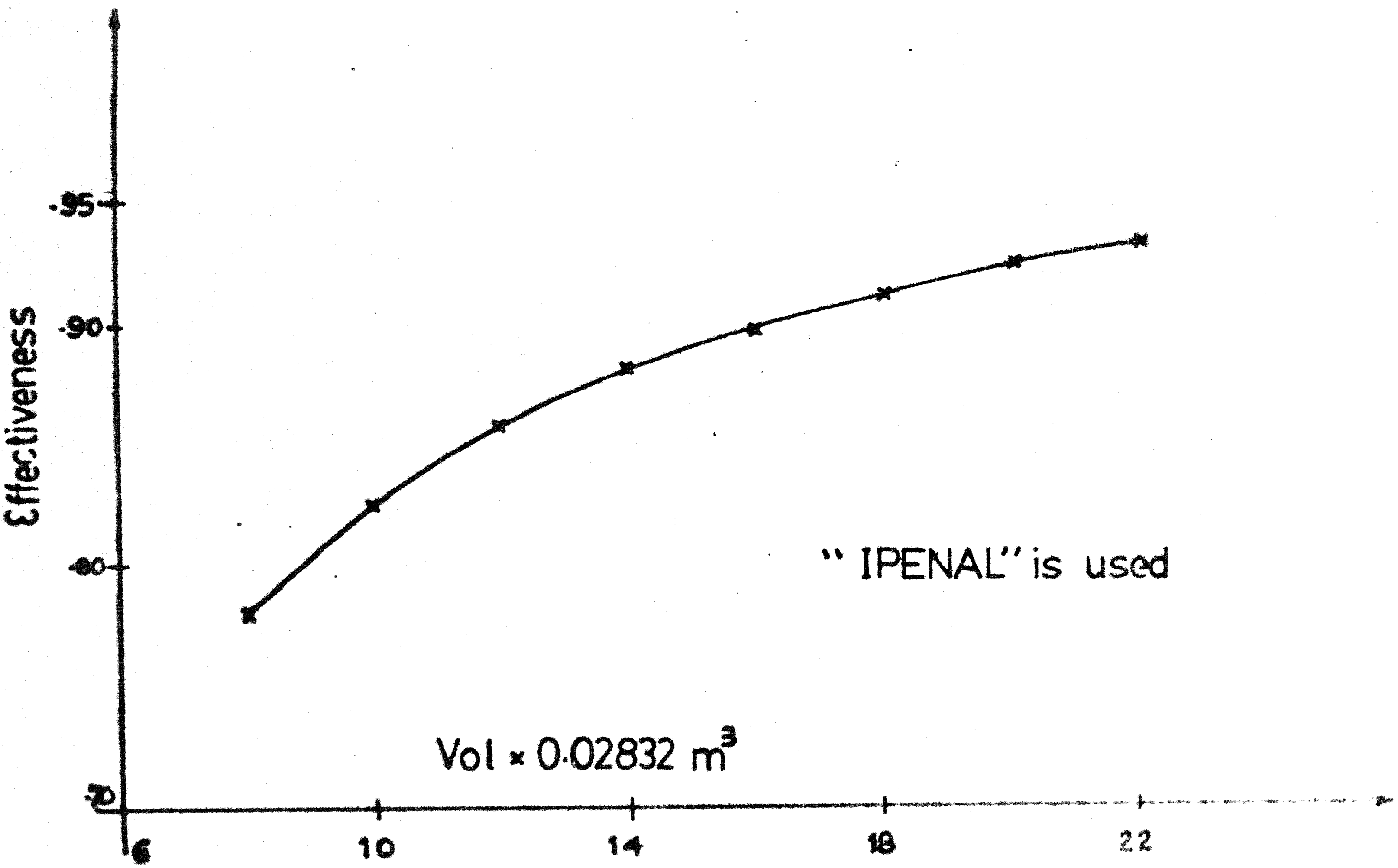


Fig 13· Variation of "ε" with Allowable Volume

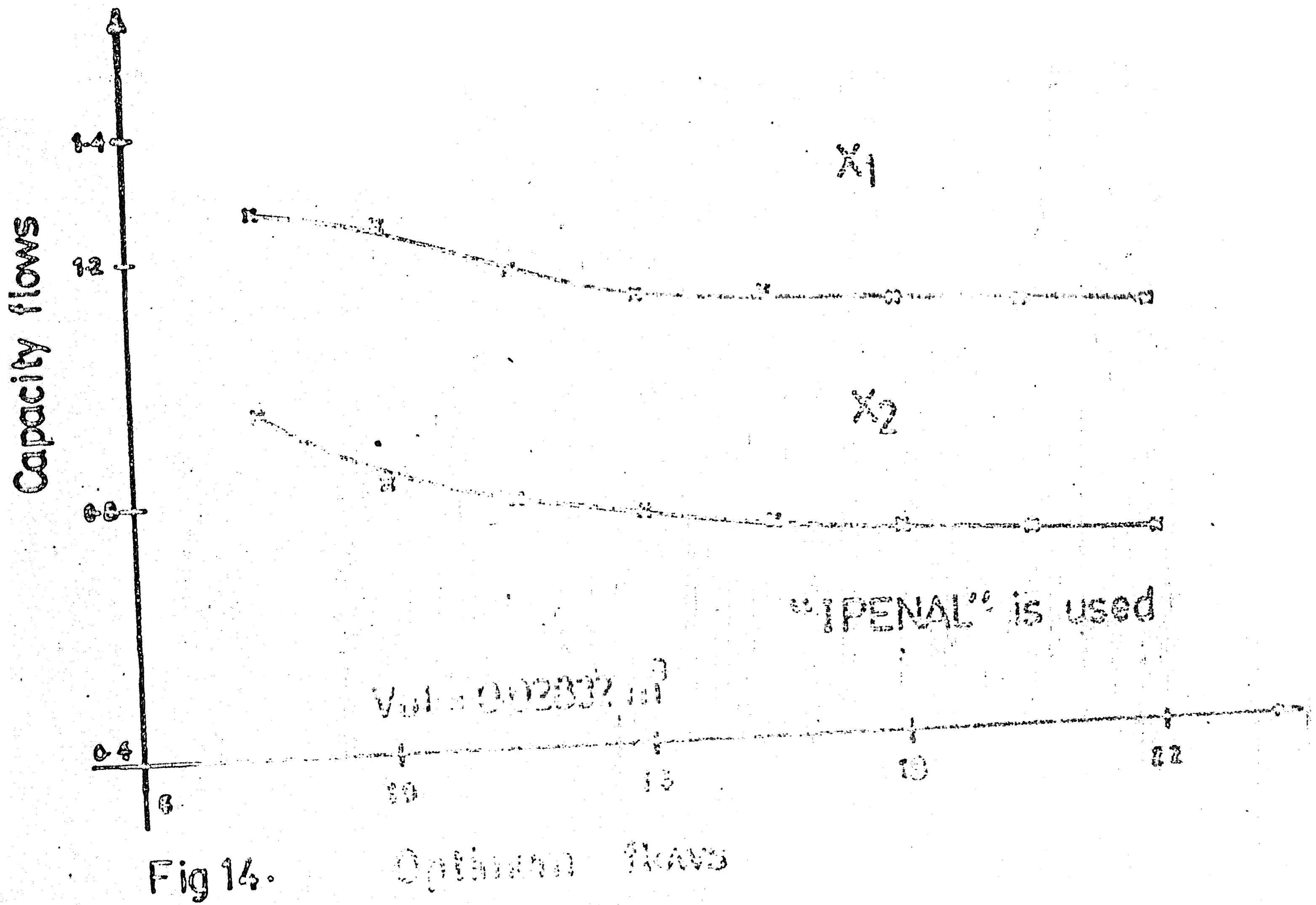
Capacity flows
1·4
1·2
0·8
0·4
X1
X2
"IPENAL" is used
Vol = 0.02832 m^3
6
10
15
19
22
Optimum flows

Fig 14.

It is obvious from Fig. 12 that the pumping power increases rapidly and the effectiveness decreases slowly as the volume decreases. The asterisk in Fig. 12 represents the pumping power and the volume corresponding to the unoptimized exchanger studied in reference [15]. For the same volume, the optimum design reduces the power Ø from 31.02 HP to 9.03 HP. If however, the pumping power requirement is kept the same, say 31.02 HP, the volume of the optimized exchanger is only 0.1614 m^3 as compared to the 0.4785 m^3, for the unoptimized case. Fig. 14, indicates the behaviour of the flow rates with volume of the heat exchanger. It is interesting to note that there is only a slight variation in the magnitude of the flow rates of both the fluids with volume.

5.2 A COMPARISON OF OPTIMIZATION METHODS

With the large number of optimization strategies available, the natural question arises - " which is the best strategy?" At present, a comparitive study of optimization methods applied to heat exchanger problems, in particular, is not available. However, several studies have been performed to evaluate the effectiveness of various optimization techniques by applying them to specially prepared test problems given by Himmelblau [2]. Most heat exchanger optimization problems are constrained and, hence, the following observations are presented for the constrained methods only.

(i) The sequential penalty approach and a gradient based search are more efficient when a large number of design variables are involved. The sequential exterior and interior penalty function methods work well with the gradient based search procedures even when using finite difference gradients. Heat exchanger performance is not generally described by a differentiable function and, therefore, internal finite difference schemes are necessary. The exterior penalty function method does not require a starting point which satisfies all the constraints. This is a distinct advantage in starting a heat exchanger design optimization for an unfamiliar application.

(ii) The method of feasible directions is rapid and effective in solving smaller problems, with perhaps upto three or four variables and having mostly linear constraints. This method, however, may have significant difficulty when a large number of design variables or nonlinear constraints are involved. Constraints on pressure drop heat transfer rate or weight, for example, are nonlinear. Geometric constraints are generally linear.

5.3 CONCLUSIONS

1. Well structured optimization programs-package has been developed and tested for it's reliability, flexibility and accuracy with standard functions, available in the open literature [1,2,3].

2. A direct transfer compact heat exchanger for use in gas turbines is designed for minimum pumping power subject to constraints on volume, dimensions, and heat transferred. The minimization problem, formulated in (13) in Chapter 4, is quite general and can be used for any heat transfer application. The ε-NTU approach is used for evaluating the heat exchanger performance. Polynomial expressions are found to approximate the experimental data for the fluid transport properties and the friction and colburn factors for the heat transfer surfaces.

3. The optimization study of the compact heat exchanger provides a design which has got much superior performance.

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APPENDIX - A : USER'S MANUAL

A.1 LIST OF SUBROUTINES

SUBROUTINES	PURPOSE
UNCONSTRAINED OPTIMIZATION :=	
UNIV	Univariate search
STEEP	Steepest descent search
CONDIR	Conjugate directions search [Powell's Method]
CONGRA	Conjugate gradient search [Fletcher - Reeves Method]
DFPM	Variable Metric Method [Davidon - Fletcher - Powell Method]
QUAD	1 - D minimization by Quadratic interpolatoin scheme
GOLD	1 - D minimization by Golden section method
CONSTRAINED OPTIMIZATION :=	
IPENAL	Interior penalty functions method
EXPEN	Exterior penalty functions method
FEAS	Method of feasible directions
START	Gets a feasible starting point
LINEAR PROGRAMMING :=	
LINEAR	Linear programming by simplex method
INTERNAL SUBROUTINES :=	
CONSTR	Determines constrains violated
GRAD , GRAD1	Package routines for determing gradient by forward differences scheme
MODIFY	Modifies the objective combining the constraints
CONVRG	Checks for convergence by perturbation schem
USER WRITTEN SUBROUTINES :=	
FUNCT	User written function subroutine
GRADU	User written gradient subroutine

A.2 COMMON STATEMENTS

Some common statements have to be included in user written programmes

1. COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
   Must be included in the mainline

2. COMMON/PENFUN/R,UF
   Insert in mainline program and function subroutine FUNCT when penalty functions are used

3. COMMON/CONVEC/G
   DIMENSION G(25)
   These statements must be included in function s subrouitne FUNCT if constraints are involved. The dimensi of G is always set to 25 in order to overcome the problem of transferring the value of G from FUNCT to internal subroutines.

4. COMMON/COMAT/DG
   DIMENSION DG(-,-)
   Placed in mainline program and used with gradient . subroutine GRADU if constraint gradients are needed as in subroutines START,FEAS .
   Matrix DG must be properly dimensioned using a appropriate integers. It's a matrix of order (NC,NN) , th elements of which represent the components of the constraints gradient.

```
A.3       EXAMPLES OF STANDARD TEST FUNCTIONS
C         -----------------------------------

C         TEST FUNCTION IS TAKEN FROM FOX [ Ref 1 ] Page 89

          COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
          COMMON/NEW/FF,ACC,IGRAD
          DIMENSION X(2)
          OPEN(UNIT=1,FILE='NN.')
          NN=2
          NC=0
          NI=0
          X(1)=0.
          X(2)=0.
          WRITE(1,5)(X(I),I=1,NN)
5         FORMAT('X(I) := ',9(1PE9.2))
          MINIM=2
          TMAX=.5
          TT=.1
          ACC=1.E-05
          NPERT=1
          LIM=15
          METHOD=1
          FF=.01
          IGRAD=1
C         TO USE GRADU , IGRAD = 2
          IOUT=4
          KFEAS=1
          CALL UNIV(MINIM,X,TT,NN,F)
          WRITE(1,10)(X(I),I=1,2),F
10        FORMAT(/' [ X ] := ',2(1PE14.6)//' [ F ] := ',1PE14.6)
          CLOSE(UNIT=1)
          STOP
          END

          SUBROUTINE FUNCT(X,F)
          DIMENSION X(2)
          F=10.*X(1)^4-20.*X(2)*X(1)^2+10.*X(2)^2+X(1)^2-2.*X(1)+5.
          RETURN
          END

          SUBROUTINE GRADJ(X,S,SUM)
          DIMENSION X(2),S(2)
          S(1)=-40.*X(1)^3+40*X(2)*X(1)-2.*X(1)+2.
          S(2)=20.*X(1)^2-20.*X(2)
          SUM=S(1)^2+S(2)^2
          IF(SUM)1,2,1
2         SUM=1.
1         RETURN
          END
          RETURN
          END
```

```
SOURCE              := FOX [ Ref : 1 ] PROBLEM F5 PP 89
NO OF VARIABLES     := TWO
CONSTRAINTS         := NONE
TEST FUNCTION       :=
MINIMIZE
        F(X)        = 10.*X(1)^4 - 20.*X(2)*X(1)^2 + 10.*X(2)^2
                    + X(1)^2 - 2.*X(1) + 5.0
NOTE                : THIS FUNCTION BELONGS TO ROSENBROCK'S BANANA
                           VALLEY FUNCTION FAMILY
STARTING POINT      :=
        [ X ]       := [ 0.0 , 0.0 ]
RESULT GIVEN        :=
        [ X ]       := [ 1. , 1. ]
          F         :=  4.0
RESULTS OBTAINED:=
BY UNIV
        [ X ]       := [ 9.9999 E-01 , 1.0000 E+00 ]
          F         :=  4.0000 E+00
BY STEEP
        [ X ]       := [ 9.8890 E-01 , 9.7736 E-01 ]
          F         :=  4.0001 E+00
BY CONDIR
        [ X ]       := [ 9.9990 E-01 , 9.9981 E-01 ]
          F         :=  4.0000 E+00

BY CONGRA
        [ X ]       := [ 8.1308 E-01 , 6.5341 E-01 ]
          F         :=  4.0355 E+00
BY DFPM
        [ X ]       := [ 9.4562 E-01 , 8.9172 E-01 ]
          F         :=  4.0030 E+00
```

```
SOURCE             := D.M.HIMMELBLAU [ Ref : 2 ] PROBLEM NO 25 PP 427
NO OF VARIABLES    := FOUR
CONSTRAINTS        := NONE
TEST FUNCTION      :=
MINIMIZE
        F(X)        = (X(1)+10.*X(2))^2 + 5.*(X(3)-X(4))^2 +
                      (X(2)-2.*X(3))^4 + 10.*(X(1)-X(4))^4
STARTING POINT     :=
        [ X ]      := [ 3.0 , -1.0 , 0.0 , 1.0 ]
RESULT GIVEN       :=
        [ X ]      := [ 0.0 , 0.0 , 0.0 , 0.0 ]
          F        :=  0.0
RESULTS OBTAINED:=
BY STEEP
        [ X ]      := [ 1.544E-01,-1.617E-02,7.949E-02,8.235E-02 ]
          F        :=  1.444E-03
BY CONDIR
        [ X ]      := [ 3.426E-02,-3.472E-03,3.635E-02,3.739E-02 ]
          F        :=  3.932E-05
BY CONGRA
        [ X ]      := [ 1.048E-02,-1.038E-02,5.859E-02,5.943E-02 ]
          F        :=  3.114E-04
BY DFPM
        [ X ]      := [ 1.585E-02,-1.661E-03,-1.457E-02,-1.418E-02 ]
          F        :=  1.056E-05
```

```
C         THIS PROBELM IS TAKEN FROM S.M.HIMMELBLAU [Ref :2] NO:24 PP426
          COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
          COMMON/PENFUN/R,UF
          COMMON/NEW/FF,ACC,IGRAD
          COMMON/CDMAT/DG
          DIMENSION X(2),DG(2,2)
          OPEN(UNIT=1,FILE='NN.')
          NN=2
          NC=2
          NI=2
          X(1)=2.0
          X(2)=2.0
          WRITE(1,5)(X(I),I=1,NN)
5         FORMAT('X(I) := ',9(1PE9.2))
          MINIM=2
          TMAX=.5
          TT=.1
          ACC=1.E-05
          NPERT=1
          LIM=15
          METHOD=5
          FF=.01
          IGRAD=1
C         TO USE GRADU , IGRADU = 2
          IOUT=6
          KFEAS=2
          FC=.1
          R=1.
          CALL IPENAL(MINIM,X,TT,NN,FC,F)
          WRITE(1,10)(X(I),I=1,NN),F
10        FORMAT(//'[ X ] := ',3(1PE14.6)//'[ F ] := ',1PE14.6)
          CLOSE(UNIT=1)
          STOP
          END

          SUBROUTINE FUNCT(X,F)
          COMMON/PENFUN/R,UF
          COMMON/CONVEC/G
          DIMENSION X(3),G(25)
          F=(X(1)-2.)^2 + (X(2)-1.)^2
          G(1)=X(1)^2 - X(2)
          G(2)=X(1) + X(2) - 2.
          UF=F
          RETURN
          END

          SUBROUTINE GRADU(X,S,SUM)
          COMMON/CDMAT/DG
          DIMENSION X(2),S(2),DG(2,2)
          S(1)=-2.*(X(1)-2.)
          S(2)=-2.*(X(2)-1.)
          SUM=S(1)^2+S(2)^2
          IF(SUM)1,2,1
2         SUM=1.
1         DG(1,1)=1.*X(1)
          DG(1,2)=-1.
          DG(2,1)=1.
          DG(2,2)=1.
          RETURN
          END
```

```
SOURCE           := D.M.HIMMELBLAU [ Ref : 2 ] PROBLEM NO 24 PP 426
NO OF VARIABLES  := TWO
CONSTRAINTS      :=
        LINEAR INEQUALITY     : ONE
        NON LINEAR INEQUALITY : ONE
TEST FUNCTION    :=
MINIMIZE
        F(X)     = (X(1)-2.)^2 + (X(2)-1.)^2
        G(1)     = X(1)^2 - X(2) <= 0.0
        G(2)     = X(1) + X(2) - 2. <= 0.0
STARTING POINT   :=
        [ X ]    := [ 2.0 , 2.0 ]
RESULT GIVEN     :=
        [ X ]    := [ 1. , 1. ]
          F      :=  1.0
RESULTS OBTAINED:=
DFPM IS USED IN ALL THE CASES
BY IPENAL
        [ X ]    := [ 1.0001 E+00 , 1.0005 E+00 ]
          F      :=  9.9968 E-01
BY EXPEN
        [ X ]    := [ 1.0001 E+00 , 1.0005 E+00 ]
          F      :=  9.9968 E-01
BY FEAS
        [ X ]    := [ 9.9795 E-01 , 9.9795 E-01 ]
          F      :=  1.0041 E+00
```

```
SOURCE             := S.S.RAO [ Ref : 3 ] PROBLEM 7.7 PP 391

NO OF VARIABLES := THREE

CONSTRAINTS        :=

        LINEAR INEQUALITY      : ONE

        NON LINEAR INEQUALITY : TWO

        BOUNDS ON INDEPENDENT
                                : THREE
               VARIABLES

TEST FUNCTION      :=

MINIMIZE

        F(X)       = X(1)^3 - 6.*X(1)^2 + 11.*X(1) + X(3)

NOTE               :   X(2) IS NOT INCLUDED IN F(X) DEFENITION

        G(1)       = X(1)^2 + X(2)^2 - X(3)^2 <= 0.0
        G(2)       = 4.0 - X(1)^2 - X(2)^2 - X(3)^2 <= 0.0
        G(3)       = X(3) - 5.0 <= 0.0
        G(4)       = -X(1) <= 0.0
        G(5)       = -X(2) <= 0.0
        G(6)       = -X(3) <= 0.0

STARTING POINT     :=

        [ X ]      := [ 0.1 , 0.1 , 3.0 ]

RESULT GIVEN       :=

        [ X ]      := [ 3.248E-08 , 1.41421 , 1.41421 ]
          F        :=   1.41422E+00

RESULTS OBTAINED:=

DFPM IS USED IN ALL THE CASES

BY IPENAL
        [ X ]      := [ 1.00E-05 , 1.41465E+00 , 1.41467E+00 ]
          F        :=   1.447821E+00
BY EXPEN
        [ X ]      := [ 5.6269E-04 , 1.40193E-02 , 1.99367E+00 ]
          F        :=   1.99368E+00

BY FEAS
        [ X ]      := [ 0.0000E+00 , 1.41422E+00 , 1.41422E+00 ]
          F        :=   1.41422E+00
```

```
SOURCE             := D.M.HIMMELBLAU [ Ref : 2 ] PROBLEM NO 11 PP 406

NO OF VARIABLES := FIVE

CONSTRAINTS        :=

        LINEAR INEQUALITY      : 6

        BOUNDS ON INDEPENDENT
                               : 10
                VARIABLES

TEST FUNCTION      :=

MINIMIZE

        F(X)       = 5.3578547*X(3)^2 + .8356891*X(1)*X(5) +37.293239*X
                   - 40792.141

NOTE               : X(2) & X(4) AREN'T INCLUDED IN F(X) DEFENITION

CONSTARINTS ARE :=

        0.0 <= 85.334407 + 0.0056858*X(2)*X(5) + 0.0006262*X(1)*X(4)
                0.0022053*X(3)*X(5) <= 92.0

        90.0 <= 80.51249 + 0.0071317*X(2)*X(5) + 0.0029955*X(1)*X(2)
                0.0021813*X(3)^2 <= 110.0

        20.0 <= 9.300961 + 0.0047026*X(3)*X(5) + 0.0012547*X(1)*X(3)
                0.0019085*X(3)*X(4) <= 25.0

                78.0 <= X(1) <= 102.0
                33.0 <= X(2) <= 45.0
                27.0 <= X(3) <= 45.0
                27.0 <= X(4) <= 45.0
                27.0 <= X(5) <= 45.0

STARTING POINT     :=

        [ X ]      := [ 78.62 , 33.44 , 31.07 , 44.18 , 35.22 ]

RESULT  GIVEN      :=

        [ X ]      := [ 78.003,29.995,45.000,36.776 ]
          F        :=  - 30665.5

RESULTS OBTAINED:=

BY DFPM
        [ X ]      := [ 78.595,33.317,30.739,44.162,35.129 ]
          F        :=  - 30493.3
```

```
C         A.4 COMPACT HEAT EXCHANGER PROBLEM
C         ----------------------------------
C
          COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
          COMMON/PENFUN/R,UF
          COMMON/NEW/FF,ACC,IGRAD
          COMMON/CALL/NCALL,VMAX,BIG,VOL
          COMMON/PARA/YY
          COMMON/EXCHGR/EF
          DIMENSION YY(5)
          OPEN(UNIT=1,FILE='R.')
          CALL RTIME(MI)
          NCALL=0
          NN=5
          NC=12
          NI=12
          YY(1)=2.5
          YY(2)=1.
          YY(3)=.5
          YY(4)=.3
          YY(5)=1.25
          VMAX=14.0
          BIG=5.
          WRITE(1,5)(YY(I),I=1,5)
5         FORMAT('[ X ] := ',9(1PE9.2))
          WRITE(1,6)BIG,VMAX
6         FORMAT('MAX FOLW RATE := ',F5.2,' V max := ',F5.2)
          MINIM=2
          TMAX=.5
          TT=.1
          ACC=1.E-05
          NPERT=1
          LIM=15
          METHOD=5
          FF=.01
          IGRAD=1
          IOUT=6
          KFEAS=1
          FC=10.
          R=10.
          CALL EXPEN(MINIM,YY,TT,NN,FC,F)
          WRITE(1,11)EF,NCALL
11        FORMAT('EFFECTIVENESS ',1PE14.6/'NO OF FUNCTION CALLS ',I5)
          WRITE(1,12)VOL
12        FORMAT('VOLUME := ',1PE14.6)
          CALL RTIME(MF)
          MT=MF-MI
          WRITE(1,10)MT
10        FORMAT('CPU TIME  := ',I5,'m.sec'/)
          CLOSE(UNIT=1)
          TYPE 100
100       FORMAT(' WANT GRAPHIC OPTION ?(Y/N) '$)
          ACCEPT 101,ANS
101       IF(ANS.NE.'Y')GOTO 102
          CALL GRAPH
C         YOU HAVE TO ASSIGN TTY 5 FOR GRAPHIC OPTION
102       STOP
          END
```

```
      SUBROUTINE FUNCT(YY,F)
C     ----------------------
C     HEAT EXCHANGER PERFORMACE ANALYSIS ROTUINE DEFINING THE
C     OBJECTIVE FUNCTION
      COMMON/PENFUN/RR,UF
      COMMON/CONVEC/G
      COMMON/CALL/NCALL,VMAX,BIG,VOL
      COMMON/EXCHGR/EF
      DIMENSION YY(5),G(25)
      REAL NUC,NTU,L
      NCALL=NCALL+1
      DO 5 I=1,5
      IF(YY(I))100,100,5
5     CONTINUE
      THI=260.
      TCI=60.
      PA=39.7
      PW=14.7
      QD=8.E+06
C     SURFACE CHAR
      RH=.00288
      ALFAH=270.
      SIGMAH=.78
      RC=.00306
      ALFAC=42.1
      SIGMAC=.129
C     FIN CHAR
      FA=.845
      DEL=.00033
      FL=.01875
      FK=100.
C     NON DIMEN
      L=VMAX*(1./3.)
      Z=QD/(THI-TCI)
C     SOLUTION
      AFRH=YY(3)*YY(5)*L^2.
      AFRC=YY(3)*YY(4)*L^2.
      VOL=YY(3)*YY(4)*YY(5)*VMAX
      E1=.75
      CH=YY(1)*Z
      CC=YY(2)*Z*10.
      CMIN=AMIN1(CH,CC)
      CMAX=AMAX1(CH,CC)
9     E=E1
      Q=CMIN*E*(THI-TCI)
      THO=THI-E*Q/CH
      TCO=TCI+E*Q/CC
      THAVG=(THI+THO)/2.
      TCAVG=(TCI+TCO)/2.
      CALL PROPC(TCAVG,RHOC,CONDC,CPC,UC)
      CALL PROPH(THAVG,RHOH,CONDH,CPH,UH)
      V1=53.345*(460.+THI)/(144.*PA)
      V2=53.345*(460.+THO)/(144.*PA)
      VM=53.345*(460.+THAVG)/(144.*PA)
      GH=CH/(CPH*SIGMAH*AFRH)
      GC=CC/(CPC*SIGMAC*AFRC)
      REC=GC*4.*RC/UC
      REH=GH*4.*RH/UH
      PRC=UC*CPC/CONDC
      PRH=UH*CPH/CONDH
C     DETERMINE COLBURN and FRICTION FACTOR FROM POLYNOMIAL FITS
      Y=ALOG10(REH*.001)
      FH=-.11998*Y^4.-.044628*Y^3.+.26677*Y^2.-.45965*Y+.58734
      COH=-.11873*Y^4.+.035861*Y^3.+.22707*Y^2.-.47894*Y-.0062232
      FH=10.^(FH)*.01
      COH=10.^(COH)*.01
      HH=COH*GH*CPH/PRH^(2./3.)
      Y=ALOG10(REC*.001)
      IF(Y.LE.0.)GOTO 1
      IF(Y.GE.1.)GOTO 2
      FC=-1.2239*Y^4.+2.1643*Y^3.-.56969*Y^2.-.87268*Y+.38629
      COC=-1.3431*Y^4.+1.6032*Y^3.+.55494*Y^2.-1.2064*Y-.083788
      GOTO 3
1     FC=.3802-.8745*Y
      COC=-.09691-Y
      GOTO 3
      COC=10.^(COC)*.01
      HC=COC*GC*CPC/PRC^(2./3.)
      FM=SQRT(2.*HH/(FK*DEL))
      Y=FM*FL
      FINEFF=TANH(Y)/Y
      EFF=1.-FA*(1.-FINEFF)
      UCOND=EFF*HH*(ALFAC/ALFAH)*HC/(ALFAC/ALFAH*HC+EFF*HH)
      NTU=ALFAH*VOL*UCOND/CMIN
      R=CMIN/CMAX
      CALL EFFECT(E1,NTU,R)
      IF(ABS(E-E1).GT..01)GOTO 9
      Y=GH/3600.
      PH=Y^2.*V1*((1.+SIGMAH^2.)*(V2/V1-1.)+FH*YY(4)*L*(VM/V1)/RH)
     1         /64.4
      Y=GC/3600.
      PC=Y^2.*FC*YY(5)*L/(RHOC*RC*64.4)
      POWER=(PH*CH*VM/CPH+PC*CC/(RHOC*CPC))/(550.*3600.)
      F=POWER
      UF=F
      EF=E1
100   G(1)=-YY(1)
      G(2)=YY(1)/BIG-1.
      G(3)=-YY(2)
      G(4)=YY(2)/(10.*BIG)-1.
      G(5)=-YY(3)
      G(6)=-YY(4)
      G(7)=-YY(5)
      G(8)=L*YY(4)-1.
      G(9)=YY(3)/(2.*YY(4))-1.
      G(10)=YY(5)/(3.*YY(3))-1.
      G(11)=VOL/VMAX-1.
      G(12)=1.-Q/QD
      RETURN
      END
      SUBROUTINE  PROPC(T,R,C,CP,U)
C     POLYNOMIAL FITS FOR TRANSPORT PROPERTIES OF AIR
      X=T/100.
      CP=.0017822*X^4.-.017893*X^3.+.067628*X^2-.091686*X+1.0348
      R=10.*(-.0019308*X^4.+.0218*X^3.-.13*X^2.+.080173*X+6.2283)
      U=.045473*X^4.-.56414*X^3.+2.6107*X^2.-5.5424*X+5.1444
      C=.1*(-.0033219*X^4.+.043706*X^3.-.29151*X^2+.88891*X+3.0051)
      RETURN
      END

      SUBROUTINE PROPH(T,R,C,CP,U)
C     POLYNOIMIAL FITS FOR TRANSPORT PROPERTIES OF WATER
      X=T/100.
      U=.1*(-.00570913*X^4.+.05755772*X^3.-.22800553*X^2 +
      1        +.6754148*X+.40343288)
      CP=.1*(.03306576*X^4.-.19582903*X^3+.30682039*X^2. +
      1        + .20886326*X +  2.36525726)
      C=.01*(.00734717*X^4.-.04445982*X^3.-.10930872*X^2. +
      1        + 2.0579495*X +1.48826838)
      RETURN
      END

      SUBROUTINE EFFECT(E,NTU,R)
C     EFFECTIVENESS FOR CROSS-FLOW EXCHANGER WITH BOTH FLUIDS UNMIXED
      REAL NTU
      SUM=(1.-EXP(-NTU))*(1.-EXP(-NTU*R))
      SUMO=SUM
      N=1
3     SUM1=1.
      SUM2=1.
      SM=1.
      DO 1 J=1,N
      SM=SM*J
      SUM1=SUM1+(NTU)^J/SM
      SUM2=SUM2+(NTU*R)^J/SM
1     CONTINUE
      SUM=SUM+(1.-EXP(-NTU)*SUM1)*(1.-EXP(-NTU*R)*SUM2)
      IF(ABS(SUM-SUMO).LT.1.E-05)GOTO 2
      N=N+1
      SUMO=SUM
      GOTO 3
2     E=SUM/(NTU*R)
      RETURN
      END
```

```
        SUBROUTINE GRAPH
C       ---------------
C
C       THIS ROUTINE USES GPGS ROUTINES FOR GRAPHIC PURPOSES
        COMMON/PARA/YY
        COMMON/TRANS/W1,W2,W3
        COMMON/EXCGHR/EFF
        COMMON/ACCEPT/CH,CC,X,Y,Z
        DIMENSION V1(6),V2(6),V3(6),W1(6),W2(6),W3(6),YY(5)
        DATA V1/0.,1.3,0.,1.,0.,1./
        DATA V2/0.,1.,0.,1.,0.,1./
        DATA V3/1.,1.3,1.,1.,1.,1./
        DATA W1/-10.,1303.,-10.,1000.,-10.,1000./
        DATA W2/-10.,900.,-10.,900.,-10.,900./
        DATA W3/901.,1204.,-10.,1000.,-10.,1000./
        CH=YY(1)*40000.
        CC=YY(2)*400000.
        X=YY(3)
        Y=YY(5)
        Z=YY(4)
        CALL NITDEV(5)
        CALL CLRDEV(5)
        CALL VPORT3(V1)
        CALL WINDW3(W1)
        TYPE 5
5       FORMAT(6(/)' WANT CAPTION ? (Y/N) '$)
        ACCEPT 6 ,CAP
6       FORMAT(A1)
        IF(CAP.NE.'Y')GOTO 7
        CALL CLRDEV(5)
        CALL TITLE
7       TYPE 40
40      FORMAT(5(/)'!! RETURN FOR CONTINUING !! '$)
        ACCEPT 15,TTY
15      FORMAT(A4)
        IF(TTY.EQ.'STOP')GOTO 100
        CALL CLRDEV(5)
        CALL MARGIN
        CALL VPORT3(V3)
        CALL WINDW3(W3)
        CALL LINE3(W3(1),W3(4),0.,0)
        CALL LINER3(0.,-W3(4),0.,1)
        WC1=W3(1)+4.0
        WC2=W3(4)-100.
        CALL SOFCTL(0)
        CALL CHAR('LIKE TO CHANGE*.')
        CALL LINE3(WC1,WC2-50.,0.,0)
        CALL CHAR('WINDW SIZE(Y/N)*.')
        ACCEPT 31,A1
31      FORMAT(A1)
        IF(A1.NE.'Y')GOTO 32
        CALL LINE3(WC1,WC2-100.,0.,0)
        CALL CHAR('GIVE ME W2(1:6)*.')
        CALL LINE3(WC1,WC2-150.,0.,0)
        ACCEPT *,(W2(I),I=1,6)
        CALL  CLRDEV(5)
32      CALL VPORT3(V1)
        CALL WINDW3(W1)
        CALL MARGIN
        CALL WINDW3(W1)
        CALL VPORT3(V2)
        CALL ROTAD(30.,2)
        CALL ROTAD(-30.,1)
        CALL OBJECT
        CALL IDEN
        CALL DIMEN
        CALL VPORT3(V3)
        CALL WINDW3(W3)
        WF=W3(4)*.6
        CALL LINE3(W3(1),WF,0.,0)
        CALL CHAR('WANT REPEAT(Y/N)*.')
        ACCEPT 12,ANS
12      FORMAT(A1)
        IF(ANS.NE.'Y')GOTO 100
        CALL VPORT3(V1)
        CALL WINDW3(W1)
        GOTO 1
100     CALL CLRDEV(5)
        STOP
```

```
      SUBROUTINE OBJECT
      COMMON/ACCEPT/CH,CC,X,Y,Z
      CALL LINE3(10.0,900.0,0.0,0)
      CALL LINER3(X,0.0,0.0,1)
      CALL LINER3(0.0,0.0,Z,1)
      CALL LINER3(-X,0.0,0.0,1)
      CALL LINER3(0.0,0.0,-Z,1)
      CALL LINER3(0.0,-Y,0.0,1)
      CALL LINER3(X,0.0,0.0,1)
      CALL LINER3(0.0,0.0,Z,1)
      CALL LINER3(0.0,Y,0.0,1)
      CALL LINER3(0.0,0.0,-Z,1)
      CALL LINER3(0.0,-Y,0.0,1)
      FIN=.025
      T=Y*FIN
C     T GIVES FIN THICKNESS
      N=IFIX(Y/(2.*T))
      CALL LINE3(10.,900.,0.,0)
      DO 10 I=1,N
      CALL LINER3(0.0,-T,0.0,0)
      CALL LINER3(X,0.0,0.0,1)
      CALL LINER3(0.0,0.0,Z,1)
      CALL LINER3(0.0,-T,0.0,0)
      CALL LINER3(0.0,0.0,-Z,1)
      CALL LINER3(-X,0.0,0.0,1)
10    CONTINUE
      TT=902.-Y
      CALL LINE3(10.0,TT,0.0,0)
      CALL LINER3(X,0.0,0.0,1)
      CALL LINER3(0.0,0.0,Z,1)
      RETURN
      END

      SUBROUTINE MARGIN
      COMMON/TRANS/W1,W2,W3
      DIMESION W1(6),W2(6),W3(6)
      CALL LINE3(W1(1),W1(4),0.,0)
      CALL LINER3(W1(2),0.,0.,1)
      CALL LINER3(0.,-W1(4),0.,1)
      CALL LINER3(-W1(2),0.,0.,1)
      CALL LINER3(0.,W1(4),0.,1)
      STOP
      END

      SUBROUTINE TITLE
      COMMON/TRANS/W1,W2,W3
      DIMENSION W1(6),W2(6),W3(6)
      DIMENSION C1(9),C2(7),C3(7)
      CALL SOFCTL(1)
      CALL CSIZEV(15)
      DATA C1/'OPTIMIZATION METHODS FOR ENGINEERING DESIGN*.'/
      DATA C2/'For COMPACT HEAT - EXCHANGERS     *.'/
      DATA C3/'Staff Advisor : Prof H.C.Agrawal*.'/
      CALL LINE3(W1(1),W1(4),0.,0)
      CALL LINER3(W1(2),0.,0.,1)
      CALL LINER3(0.,W1(4),0.,1)
      CALL LINER3(-W1(2),0.,0.,1)
      CALL LINER3(0.,W1(4),0.,1)
      WI=200.
      WW=100.
      WY=W1(4)-200.
      CALL LINE3(WW,WY,0.,0)
      CALL CHAR(C1,45)
      CALL LINE3(WW,WY-200.,0.,0)
      CALL CHAR(C2,31)
      CALL LINE3(WW,WY-400.,0.,0)
      CALL LINE3(WW,WY-400.,0.,0)
      CALL CHAR(C3,34)
      RETURN
      END
```

```
SUBROUTINE DIMEN
COMMON/TRANS/W1,W2,W3
COMMON/ACCEPT/CH,CC,X,Y,Z
COMMON/EXCGHR/EFF
DIMENSION W1(6),W2(6),W3(6)
CALL SOFCTL(1)
CALL LINE3(W2(1)+10.,W2(4)-50.,0.,0)
CALL CSIZEH(30)
CALL CSIZEV(18)
CALL CSHEA(.2)
CALL CHAR('OPTIMIZED DIMENSIONS*.')
CALL SOFCTL(0)
D1=W2(2)*.5
D2=W2(4)-200.
CALL LINE3(D1,D2,0.,0)
CALL CHAR('WIDTH :=*.')
CALL CHARF(X,7,2)
CALL LINE3(D1,D2-50.,0.,0)
CALL CHAR('BREADTH :=*.')
CALL CHARF(Y,7,2)
CALL LINE3(D1,D2-100.,0.,0)
CALL CHAR('HEIGHT :=*.')
CALL CHARF(Z,8,2)
CALL LINE3(D1,D2-150.,0.,0)
CALL CHAR('AIRFLOW :=*.')
CALL CHARF(CH,9,1)
CALL LINE3(D1,D2-200.,0.,0)
CALL CHAR('WATER FLOW :=*.')
CALL CHARF(CC,9,1)
CALL LINE3(D1,D2-250.,0.,0)
CALL CHAR('EFFECTIVENESS :=*.')
CALL CAHRF(EFF,6,4)
RETURN
END
```

RESULT OF TRADE-OFF STUDIES

VOLUME	METHOD	F	X1	X2	E
8	IPENAL	21.684	1.2745	0.9481	0.7842
	EXPEN	21.548	1.2870	0.8509	0.7763
	FEAS	21.700	1.2170	0.9540	0.7842
10	IPENAL	15.173	1.2106	0.8171	0.8251
	EXPEN	15.162	1.2116	0.8188	0.8243
	FEAS	15.400	1.1940	0.9520	0.8356
12	IPENAL	11.723	1.1648	0.7949	0.8571
	EXPEN	11.716	1.1650	0.7943	0.8570
	FEAS	12.030	1.1440	0.9550	0.8710
14	IPENAL	9.592	1.1341	0.6777	0.8800
	EXPEN	9.606	1.1286	0.8039	0.8840
	FEAS	9.770	1.1170	0.8990	0.8920
16	IPENAL	8.155	1.1123	0.7423	0.8971
	EXPEN	8.126	1.1111	0.7389	0.8969
	FEAS	8.550	1.0880	0.9610	0.9153
18	IPENAL	7.125	1.0947	0.7284	0.9114
	EXPEN	7.103	1.0946	0.7190	0.9503
	FEAS	6.190	1.0480	0.9670	0.9503
20	IPENAL	6.351	1.0812	0.7139	0.9227
	EXPEN	6.334	1.0819	0.6995	0.9212
	FEAS	6.781	1.0580	0.9650	0.9412
22	IPENAL	5.750	1.0704	0.7011	0.9319
	EXPEN	5.736	1.0718	0.6814	0.9299
	FEAS	5.723	1.0480	0.9667	0.9502

APPENDIX - B : PROGRAMS PACKAGE LISTINGS

```
C         ==============================================================
          SUBROUTINE RANGE(NN,NQUIT,NREP,TT,TT1,X,S,AF,AL,XH,XL,Z)
C         ==============================================================
C         THIS SUBROUTINE DETERMINES THE RANGE WHICH CONTAINS THE MINIMUM
C         NQUIT=1 RANGE FOR MINIMUM LOCATED
C         NQUIT=2 CONSTRAINT IN EFFECT
C         NQUIT=3 NO IMPROVEMENT  POSSIBLE IN THE OBJECTIVE FUNCTION
C         NREP=1 NO.OF ITERATIONS INADEQUATE TO ESIMATE THE RANGE
C         NREP=2 RANGE ESTIMATED
C         NREP=3 NO IMPROVEMENT IN OBJECTIVE FUNCTION POSSIBLE
C         TT,TT1 = INITIAL STEP SIZE
C         TMAX = MAX STEP SIZE ALLLOWED
C         AL,AF= COORD.AND FUNCTION VALUES FOR  QUAD.INTERPOLATION
C         Z=INITIAL AND FINAL VALUES ALONG A SEARCH DIRECTION
C         KFEAS=1 UNCONSTRAINED OR EXTERIOR PENALTY FUNCTION PROBLEM
C         KFEAS=2 UNCONSTRAINED PROBLEM USING INTERIOR PENALTY FUNCTION
C         KFEAS=3 CONSTRAINED PROBLEM USING FEASIBLE DIRECTION METHOD
C         METHOD -- CHOICE OF MULTIVARIATE SEARCH TECHNIQUE
C         METHOD=1 UNIVARIATE SEARCH
C         METHOD=2 STEEPEST DESCENT
C         METHOD=3 CONJUCATE DIRECTIONS [ POWELL'S METHOD ]
C         METHOD=4 CONJUCATE GRADIENT [ FLETCHER REEVES METHOD ]
C         METHOD=5 VARIABLE METRIC  [ DAIVDON FLETCHER POWELL METHOD ]
C         IOUT -- TYPE OF OUTPUT DESIRED
C         NOTE := HIGHER ORDERS HAVE HIGHLY SUPPRESSED BUT
C         IMPORTANT RESULTS
C         IOUT=1 OUTPUT AT ALL STAGES
C         IOUT=2 OUTPUT AFTER EACH 1D MINIMIZATIONS
C         IOUT=3 FINAL OUTPUT OF UNCONSTAINED MINIMIZATION
C         IOUT=4 OUTPUT AT ALL STAGES CONSTRAINED MINIMIZATION
C         IOUT=5 FINAL OUTPUT OF EACH CONSTRIANED MINIMIZATION
C         IOUT=6 ONLY FINAL OUTPUT OF CONSTRAINED MINIMIZATION
          ..............................................................
          COMMON/MIN/AAL
          COMMON/CONVEC/G
          COMMON/INCOVC/GO
          COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
          COMMON/PENFUN/R,UF
          COMMON/NEW/FF,ACC,IGRAD
          COMMON/ALARM/KAL,JAW
          DIMENSION X(NN),S(50),AF(3),AL(3),Z(2)
          DIMENSION YX(25),G(25),GO(25)
          DIMENSION DG(25,25)
          KNA=1
32        AAL=TT
          IF(IOUT.GT.2)GOTO 555
          WRITE(1,305)KAL,JAW
          WRITE(1,300)
          WRITE(1,3)(X(J),J=1,NN)
          WRITE(1,303)Z(1)
          WRITE(1,304)(S(J),J=1,NN)
305       FORMAT(/35('-')/'ITERATION NO ',I2,' CYCLE NO ',I2/35('-'))
300       FORMAT('INITIAL VALUES OF X(J)')
303       FORMAT('INTIAL VALUE OF THE OBJECTIVE FUNCTION'/1PE14.6)
304       FORMAT('INITIAL VALUES OF S(J)'/9(1PE14.6))
          IF(NC)555,555,577
577       WRITE(1,572)
572       FORMAT('INTIAL VALUES OF THE CONSTRAINTS')
          WRITE(1,3)(GO(J),J=1,NC)
555       H=Z(1)
          IF(NC)570,570,571
571       DO 596 J=1,NC
596       G(J)=GO(J)
          IF(KFEAS-2)590,591,591
591       CALL CONSTR(NPENAL)
          IF(NPENAL-1)590,590,592
592       WRITE(1,593)KAL,JAW
593       FORMAT('*  X(J)1 INFEASIBLE IN',I3,'th ITERATION OF '
     1    ,I2,' CYCLE *')
          WRITE(1,3)(GO(J),J=1,NC)
595       CALL START(X,NN,NC.TT,DG,IOUT)
590       CONTINUE
          IF(KFEAS-2)581,581,570
581       IF(IOUT-1)582,582,569
582       WRITE(1,583)
583       FORMAT(/'NPENAL',2X,'KNN',6X,'STEP SIZE',8X,'UNAUGMETED',
     18X,'FUNCTION',16X,'DESIGN VECTOR')
          GOTO 569
570       IF(IOUT-1)568,568,569
568       WRITE(1,562)
```

```
KPE=1
KZ=1
AL(1)=0.
AF(1)=Z(1)
KZ=KZ+1
KN=1
KK=1
KNN=1
KZ1=1
IF(KZ1-LIM)23,23,24
TT=TT1
CHECK IF NO.OF ITERATIONS XDS THE MAX. SPECIFIED.
KPENAL=1. MAX NO.OF ITERATIONS INADEQUATE TO LOCATE THE MIN.
KPENAL=2. CONSTRAINT VIOLATED AFTER INITIAL DECREASE IN FUNCN.
KPENAL=3. CONSTRAINT VIOLATED RIGHT FROM START.
GOTO(102,22,122)KPENAL
DO 28 J=1,NN
X(J)=YX(J)
IF(NC)514,514,551
DO 103 J=1,NC
GO(J)=G(J)
GOTO 514
DO 26 J=1,NN
X(J)=X(J)+AALO*S(J)
IF(NC)514,514,573
DO 574 J=1,NC
G(J)=GO(J)
GOTO 514
DO 13 J=1,NN
YX(J)=X(J)+AAL*S(J)
CALL FUNCT(YX,F)
CALL MODIFY(F)
CALL CONSTR(NPENAL)
IF(IOUT-1)556,556,557
IF(NC)584,584,585
IF(KFEAS-2)587,587,584
WRITE(1,586)NPENAL,KNN,AAL,UF,F,(YX(J),J=1,NN)
FORMAT(2(I4,2X),3(1PE14.6,2X),5(5(1PE14.6),/,60X))
GOTO 557
WRITE(1,7)NPENAL,KNN,AAL,F,(YX(J),J=1,NN)
FORMAT(X,I4,2X,I4,2X,1PE14.6,2X,1PE14.6,2X,5(6(1PE14.6),/44X))
IF(NPENAL-1)60,60,61
IF(KNN-1)62,62,63
KPENAL=3
IF(KFEAS-2)64,64,65
TT=.5*TT
AAL=TT
KZ1=KZ1+1
GOTO 16
EXTRAPOLATION SCHEME TO LOCATE THE CONSTRAINT.
IF(F-H)66,66,64
DO 67 K=1,NI
IF(G(K))67,68,68
CONTINUE
D=G(K)
MC=K
DO 69 K=1,NC
IF(G(K))69,69,70
IF(G(K)-D)69,69,71
D=G(K)
MC=K
CONTINUE
AAL=(GO(MC)*AAL-G(MC)*H1)/(GO(MC)-G(MC))
DO 80 J=1,NN
YX(J)=X(J)+AAL*S(J)
CALL FUNCT(YX,F)
CALL MODIFY(F)
CALL CONSTR(NPENAL)
IF(IOUT-1)558,558,559
IF(KFEAS-2)588,588,589
WRITE(1,586)NPENAL,KNN,AAL,UF,F,(YX(J),J=1,NN)
GOTO 559
WRITE(1,7)NPENAL,KNN,AAL,F,(YX(J),J=1,NN)
IF(G(MC)-.1E-10)72,73,74
IF(ABS(G(MC))-ACC)73,73,75
IF(NPENAL-1)76,76,74
TT=.5*(AAL-AALO)
KPE=2
GOTO 60
IF(NPENAL-1)78,78,74
IF(KZ1-LIM)77,77,24
```

```
      DO 104 J=1,NC
104   GO(J)=G(J)
      Z(1)=F
      GOTO 122
63    KPENAL=2
      IF(KFEAS-2)82,82,83
C     STEP SIZE  CHANGE. CONSTRAINT VIOLATED AFTER INITIAL DECREASE
C     OF OBJECTIVE FUNCTION.
82    TT=AAL-AALO
      AAL=AAL-TT
      TT=.5*TT
      AAL=AAL+TT
      KPE=2
      KZ1=KZ1+1
      GOTO 15
83    IF(F-H)66,66,82
60    AALO=AAL
      KPENAL=1
      Z(2)=F
      IF(Z(2)-H)14,14,15
14    H=Z(2)
      H1=AAL
      IF(NC)552,552,553
553   DO 90 J=1,NC
90    GO(J)=G(J)
C     DETERMINATION AND UPDATING OF AF AND AL. COMPARISON WITH MAX.
C     STEP SIZE SPECIFIED.
552   IF(KZ-3)530,530,531
530   AF(KZ)=Z(2)
      AL(KZ)=AAL
      KZ=KZ+1
      GOTO 532
531   AF(1)=AF(2)
      AF(2)=AF(3)
      AF(3)=Z(2)
      AL(1)=AL(2)
      AL(2)=AL(3)
      AL(3)=AAL
532   CONTINUE
      IF(KPE-1)49,49,48
49    IF(KNN-3)536,537,537
536   TT=AAL*(KNN+1)
      AAL=TT
      GOTO 50
537   AAL=2.*AAL
50    ST=AAL-AALO
      IF(ST-TMAX)51,51,52
52    ST=TMAX
      AAL=AALO+ST
      GOTO 51
48    AAL=AAL+TT
51    CONTINUE
      KNN=KNN+1
      KK=KK+1
      KZ1=KZ1+1
      GOTO 15
C     REVERSAL OF SEARCH DIRECTION. DECREASE OF STEP SIZE.
15    IF(KK-1)18,18,17
18    IF(KN-1)46,46,47
46    GOTO(20,47,20,47,47),METHOD
20    DO 19 J=1,NN
19    S(J)=-S(J)
      KN=KN+1
      GOTO 15
17    IF(KZ-3)533,533,534
533   AF(KZ)=F
      AL(KZ)=AAL
      GOTO 535
534   AF(1)=AF(2)
      AF(2)=AF(3)
      AF(3)=F
      AL(1)=AL(2)
      AL(2)=AL(3)
      AL(3)=AAL
535   XL=AL(1)
      XH=AL(3)
      NREP=2
      NQUIT=1
      IF(TT-TT1)500,500,502
502   TT=TT1
      GOTO 500
47    IF(KNA-5)151,151,152
```

```
      KNA=KNA+1
      GOTO 16
      NQUIT=2
122   TT=TT1
      IF(NC)153,153,575
      DO 576 J=1,NC
575   G(J)=GO(J)
576   GOTO 153
      NREP=1
514   NQUIT=1
      IF(TT-TT1)500,500,503
      TT=TT1
503   GOTO 500
      NQUIT=3
152   IF(NC)153,153,579
      DO 580 J=1,NC
579   G(J)=GO(J)
580   Z(2)=Z(1)
153   NREP=3
      F=Z(1)
      FORMAT(9(1PE14.5))
3     IF(IOUT.GT.2)GOTO 561
500   WRITE(1,501)
560   FORMAT('VALUES OF S(J) AT EXIT FROM RANGE ARE')
501   WRITE(1,3)(S(J),J=1,NN)
      WRITE(1,85)NREP,NQUIT
      FORMAT(2X,'NREP',2X,'NQUIT'/3X,I2,5X,I2)
85    WRITE(1,92)(G(J),J=1,NC)
      FORMAT('G FROM RANGE AT EXIT'/9(1PE14.6))
92    RETURN
561   END
```

```
C     =====================================================================
      SUBROUTINE QUAD(X,S,NN,AL,AF,F)
C     =====================================================================
C     COMPUTATION OF MINIMUM USING QUADRATIC INTERPOLATION
C     X -- DESIGN VECTOR
C     NN -- ORDER OF THE DESIGN VECTOR
C     ACC -- ACCURACY DESIRED
C     S -- SEARCH DIRECTION VECTOR
C     F -- FUNCTION VALUE AT THE MINIMUM
C     LIM -- MAX NO. OF ITERATIONS
C     AL -- COORDINATES OF  THREE POINTS
C     AF -- FUNCTION VALUES AT THESE THREE POINTS
C     ......................................................................
      COMMON/PENFUN/R,UF
      COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
      COMMON/MIN/AAL
      COMMON/NEW/FF,ACC,IGRAD
      COMMON/ALARM/KAL,JAW
      DIMENSION  X(NN),S(50),AL(3),AF(3)
      DIMENSION ZZ(25)
      IF(IOUT-1)44,44,45
44    WRITE(1,46)
46    FORMAT('RANGE RESULTS FOR QUADRATIC APROXIMATION')
      WRITE(1,3)(AF(J),J=1,3)
      WRITE(1,3)(AL(J),J=1,3)
3     FORMAT(3(1PE14.5))
      IF(NC)78,78,79
78    WRITE(1,43)
43    FORMAT('RESULTS FROM QUDRATIC INTERPOLATION'/2X,'NO',4X,
     1'STEPSIZE',9X,'FUNCTION',11X,'DESIGN VECTOR'/)
      GOTO 45
79    IF(KFEAS-2)80,80,78
80    WRITE(1,81)
81    FORMAT(/2X,'NO',4X,'STEP SIZE',7X,'UNAUGMENTED',7X,'FUNCTION',
     1       12X,'DESIGN VECTOR')
45    CONTINUE
      KC1=1
      KC2=1
      NUM=1
      H=AF(2)
2     AAL=( AF(1)*(AL(2)^2-AL(3)^2) + AF(2)*(AL(3)^2-AL(1)^2) +
     1 AF(3)*(AL(1)^2-AL(2)^2))/(2.*( AF(1)*(AL(2)-AL(3)) +
     1 AF(2)*(AL(3)-AL(1)) + AF(3)*(AL(1)-AL(2))))
      DO 6 J=1,NN
6     ZZ(J)=X(J)+AAL*S(J)
      CALL FUNCT(ZZ,F)
      CALL MODIFY(F)
      IF(IOUT-1)76,76,77
76    IF(NC)82,82,83
82    WRITE(1,52)NUM,AAL,F,(ZZ(K),K=1,NN)
52    FORMAT(2X,I2,2(1PE14.6),4(7(1PE14.6),/,32X))
      GOTO 77
83    IF(KFEAS-2)84,84,82
84    WRITE(1,85)NUM,AAL,UF,F,(ZZ(I),I=1,NN)
85    FORMAT(2X,I2,3(1PE14.6),8(6(1PE14.6)/46X))
C     RE-ESTIMATION OF THE RANGE LIMITS, SHUFFLING OF COORDINATES OF
C     THE THREE POINTS.
77    IF(ABS(F-H).LE.ACC)GOTO 7
8     IF(NUM-LIM)14,15,15
14    NUM=NUM+1
      IF(AF(2)-F)9,55,10
9     H=AF(2)
      IF(AAL-AL(2))17,7,18
17    AF(1)=F
      AL(1)=AAL
      IF(KC2-3)23,25,25
23    KC1=1
      KC2=KC2+1
      GOTO 2
18    AF(3)=F
      AL(3)=AAL
      IF(KC1-3)24,21,21
24    KC2=1
      KC1=KC1+1
      GOTO 2
10    H=F
      IF(AAL-AL(2))11,7,12
11    AF(3)=AF(2)
      AF(2)=F
      AL(3)=AL(2)
      AL(2)=AAL
```

```
12        AF(1)=AF(2)
          AF(2)=F
          AL(1)=AL(2)
          AL(2)=AAL
          IF(KC2-3)22,25,25
22        KC1=1
          KC2=KC2+1
          GOTO 2
C         DETERMINITATION OF NEW COORDINATES FOR THE FIRST POINT FOR
C         FOR FASTER CONVERGENCE
21        IF(IOUT.EQ.1)WRITE(1,92)
92        FORMAT('* CONVERGENCE SCHEME CALLED FROM 21 *')
          TLT=AL(2)-AL(1)
          TX=AL(3)-AL(2)
          KNN=1
          ST1=0.
30        ST=KNN*TX
          ST1=ST1+ST
          IF(ST1-TLT)41,2,2
41        AAL=AL(2)-ST
          DO 27 I=1,NN
27        ZZ(I)=X(I)+AAL*S(I)
          CALL FUNCT(ZZ,F)
          CALL MODIFY(F)
          IF(F-AF(2))28,28,42
28        AF(2)=F
          AL(2)=AAL
          KNN=KNN+1
          GOTO 30
42        AF(1)=F
          AL(1)=AAL
          GOTO 2
C         DETERMINATION OF NEW COORDINATES FOR THE THIRD POINT FOR
C         FASTER CONVERGENCE.
25        IF(IOUT.EQ.1)WRITE(1,93)
93        FORMAT('* CONVERGENCE SCHEME CALLED FROM 25 *')
          TLT=AL(3)-AL(2)
          TX=AL(2)-AL(1)
          KNN=1
          ST1=0.
34        ST=KNN*TX
          ST1=ST1+ST
          IF(ST1-TLT)39,2,2
39        AAL=AL(2)+ST
          DO 31 I=1,NN
31        ZZ(I)=X(I)+AAL*S(I)
          CALL FUNCT(ZZ,F)
          CALL MODIFY(F)
          IF(F-AF(2))32,32,40
32        AF(2)=F
          AL(2)=AAL
          KNN=KNN+1
          GOTO 34
40        AF(3)=F
          AL(3)=AAL
          GOTO 2
15        IF(IOUT.GT.2)GOTO 7
          WRITE(1,1) LIM,KAL
1         FORMAT('* ACCCURACY NOT ACHIEVED IN ',I2,' ITERATIONS OF'
         1          ,I2,' th 1.D CYCLE *')
          GOTO 88
7         IF(IOUT.GT.2)GOTO 88
          WRITE(1,89)NUM
89        FORMAT('CONVERGENCE ACHIEVED IN ',I2,' ITERATIONS')
          GOTO 57
55        NMIN=NUM-1
C         ALTER HERE
57        WRITE(1,90)AF(2),F
90        FORMAT(5X,'AF(2)',10X,'F'/2(1PE14.6))
88        IF(AF(2)-F)35,35,38
35        F=AF(2)
          AAL=AL(2)
          DO 37 I=1,NN
37        ZZ(I)=X(I)+AL(2)*S(I)
          IF(NCD)38,38,150
150       CALL FUNCT(ZZ,F)
          CALL MODIFY(F)
38        DO 75 I=1,NN
75        X(I)=ZZ(I)
          IF(IOUT.GT.2)RETURN
          WRITE(1,95)AAL,AL(2),(ZZ(I),I=1,NN)
```

```
      SUBROUTINE GOLD(X,S,NN,XL,XH,F)
C     ===================================================================
C     DETERMINATION OF MINIMUM IN SEARCH DIRECTION USING
C     GOLDEN SECTION METHOD
C     X -- DESIGN VECTOR
C     NN -- ORDER OF DESIGN VECTOR
C     S -- SEARCH DIRECTION VECTOR
C     XL -- LOWER LIMIT FOR MINIMUM
C     XH -- HIGHER LIMIT FOR MINIMUM
C     LIM -- MAX NUMBER OF ITERATIONS PERMITTED
C     F -- FUNCTION VALUE AT THE MINIMUM
C     ACC -- ACCURACY DESIRED
C     ...................................................................
      COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
      COMMON/PENFUN/R,UF
      COMMON/MIN/AAL
      COMMON/NEW/FF,ACC,IGRAD
      COMMON/ALARM/KAL,JAW
      DIMENSION X(NN),S(50),AL(3),YY(3)
      DIMENSION ZZ(25)
      IF(IOUT-1)37,37,38
37    WRITE(1,39)
39    FORMAT(/'GOLDEN SECTION RESULTS'/)
      IF(NC)85,85,86
85    WRITE(1,40)
40    FORMAT(/2X,'STEP SIZE',9X,'FUNCTION',10X,'DESIGN VECTOR')
      GOTO 38
86    IF(KFEAS-2)87,87,85
87    WRITE(1,88)
88    FORMAT(/2X,'STEP SIZE',6X,'UNAUGMENTED',4X,'FUNCTION',13X,
     1'DESIGN VECTOR')
38    CONTINUE
      KZ=1
61    B=XH-XL
      AL(1)=XL+.618*B
      AL(2)=XH-.618*B
      DO 71 J=1,2
      DO 70 K=1,NN
70    ZZ(K)=X(K)+ABS(AL(J))*S(K)
      CALL FUNCT(ZZ,F)
      CALL MODIFY(F)
      YY(J)=F
      IF(IOUT-1)80,80,71
80    IF(NC)89,89,90
89    WRITE(1,52) AL(J),YY(J),(ZZ(K),K=1,NN)
52    FORMAT(2D14.6,4(7D14.6,/,28X))
      GOTO 71
90    IF(KFEAS-2)92,92,89
92    WRITE(1,91) AL(J),UF,YY(J),(ZZ(K),K=1,NN)
91    FORMAT(3D14.6,5(6D14.6,/,42X))
71    CONTINUE
      IF(ABS(YY(2)-YY(1))-ACC)15,15,16
16    IF(KZ-LIM)25,26,26
25    KZ=KZ+1
C     DETERMINATION OF NEW LIMITS OF RANGE
      IF(YY(2)-YY(1))18,15,17
18    XH=AL(1)
      YY(1)=YY(2)
      AL(1)=AL(2)
      J=2
      GOTO 31
17    XL=AL(2)
      YY(2)=YY(1)
      AL(2)=AL(1)
      J=1
31    B=XH-XL
      IF(J-1)32,32,33
32    AL(1)=XL+.618*B
      DO 34 K=1,NN
34    ZZ(K)=X(K)+AL(1)*S(K)
      GOTO 35
33    AL(2)=XH-.618*B
      DO 36 K=1,NN
36    ZZ(K)=X(K)+AL(2)*S(K)
35    CALL FUNCT(ZZ,F)
      CALL MODIFY(F)
      YY(J)=F
      IF(IOUT-1)84,84,85
84    IF(NC)93,93,94
93    WRITE(1,52) AL(J),YY(J),(ZZ(K),K=1,NN)
      GOTO 45
94    IF(KFEAS-2)95,95,93
95    WRITE(1,91) AL(J),UF,YY(J),(ZZ(K),K=1,NN)
```

```
          J=3
C         COMPUTATION OF THE MID-POINT TOO CONFIRM MINIMUM AND
C         SATISFICATION FO ACCURACY CRITERION
          DO 72 K=1,NN
 72       ZZ(K)=X(K)+ABS(AL(3))*S(K)
          CALL FUNCT(ZZ,F)
          CALL MODIFY(F)
          YY(J)=F
          IF(IOUT-1)82,82,83
 82       IF(NCD)96,96,97
 96       WRITE(1,52) AL(J),YY(J),(ZZ(K),K=1,NN)
          GOTO 83
 97       IF(KFEAS-2)98,98,96
 98       WRITE(1,91) AL(J),UF,YY(J),(ZZ(K),K=1,NN)
          IF(ABS(YY(3)-YY(1))-ACC)19,19,20
 20       XL=AL(2)
          XH=AL(1)
          GOTO 61
 19       AAL=ABS(AL(3))
          GOTO 27
 26       IF(YY(2)-YY(1))76,76,77
 76       YY(3)=YY(2)
          J=2
          GOTO 78
 77       YY(3)=YY(1)
          J=1
 78       DO 79 K=1,NN
 79       ZZ(K)=X(K)+AL(J)*S(K)
          WRITE(1,1) LIM,KAL
 1        FORMAT('* ACCURACY NOT ACHIEVED IN ',I3,' ITERATIONS
          1        OF ',I3,' tn 1.D CYCLE *')
 27       DO 75 J=1,NN
 75       X(J)=ZZ(J)
          F=YY(3)
          RETURN
          END
```

```
C     ==================================================================
      SUBROUTINE UNIV(MINIM,X,TT,NN,F)
C     ==================================================================
C     X -- DESIGN VECTOR
C     NN -- ORDER OF THE DESIGN VECTOR
C     MINIM -- CHOICE OF 1.D SEARCH TECHNQUE
C     MINIM=1 GOLDEN SECTION
C     MINIM=2 QUADRATIC INERPOLATION
C     TT -- INITIAL STEP SIZE
C     ACC -- ACCURACY DESIRED
C     F -- OPTIMUM VALUE OF OBJECTIVE FUNCTION
C     ..................................................................
      COMMON/INCDVC/GD
      COMMON/CONVEC/G
      COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,IOUT,NPERT
      COMMON/NEW/FF,ACC,IGRAD
      COMMON/ALARM/KAL,JAW
      DIMENSION X(NN),AL(3),AF(3),Z(2)
      DIMENSION S(50),G(25),GD(25)
      KTK=1
      TT1=TT
      KAL=0
      CALL FUNCT(X,F)
 86   CALL MODIFY(F)
      Z(1)=F
      IF(NC)77,77,78
 78   DO 79 J=1,NC
 79   GD(J)=G(J)
 77   CONTINUE
 27   N=1
 22   KAL=KAL+1
      S(N)=1.
      DO 11 J=1,NN
      IF(J-N)12,11,12
 12   S(J)=0.
 11   CONTINUE
 87   CALL RANGE(NN,NQUIT,NREP,TT,TT1,X,S,AF,AL,XH,XL,Z)
      GOTO (21,30,31)NREP
 31   GOTO (30,21,21)NQUIT
 30   GOTO (900,901)MINIM
 900  CALL GOLD(X,S,NN,XL,XH,F)
      GOTO 903
 901  CALL QUAD(X,S,NN,AL,AF,F)
 903  IF(NC)80,80,81
 81   DO 82 J=1,NC
 82   GD(J)=G(J)
 80   Z(2)=F
      IF(KTK-3)83,84,84
 83   IF(ABS(Z(2)-Z(1))-ACC)84,84,21
 21   N=N+1
      Z(1)=Z(2)
      IF(N-NN)22,22,26
 26   IF(NN-1)20,20,32
 32   IF(NQUIT-3)27,20,20
C     CONVERGENCE RULE
 84   IF(NPERT-1)20,20,85
 85   CALL CONVRG(X,S,Z,NN,NC,KTK,HK,F)
      GOTO (27,86,187,20)KTK
 187  KAL=KAL+1
      GOTO 87
C     PRINT OUTPUT
 20   F=Z(2)
      IF(IOUT.GT.4)GOTO 75
      WRITE(1,3)
 3    FORMAT('POWEL"s UNIVARIATE METHOD USED ')
 75   IF(IOUT.GT.5)GOTO 76
      WRITE(1,177)KAL
 177  FORMAT('NO OF 1.D CYCLES ',I3)
      IF(IOUT.GT.3)GOTO 76
      WRITE(1,211) (X(J),J=1,NN)
 211  FORMAT('[ X ] := ',9(1PE14.6))
      WRITE(1,4) Z(2),ACC
 4    FORMAT('[ F ] := ',1PE14.6//'ACCURACY ACHIEVED ',F9.5)
 76   F=Z(2)
      TT=TT1
      RETURN
      END
```

```
C     ========================================================================
      SUBROUTINE STEEP(MINIM,X,TT,NN,F)  !! STEEPEST DESCENT METHOD !!
C     ========================================================================
C     X -- DESIGN VECTOR
C     NN -- ORDER OF THE DESIGN VECTOR
C     MINIM -- CHOICE OF 1.D SEARCH TECHNIQUE
C     MINIM=1 GOLDEN SECTION
C     MINIM=2 QUADRATIC INTERPOLATION
C     TT -- INITIAL STEP SIZE
C     ACC -- ACCURACY DESIRED
C     FF -- PERCENTAGE CHANGE IN VARIABLE DESIRED FOR FINITE NCE
C     DIFFERENCE COMPUTATION OF THE GRADIENTS
C     IGRAD -- CHOICE OF GRADIENT TECHNIQUE
C     IGRAD=1 USE PACKAGE ROUTINE
C     IGRAD=2 USE USER ROUTINE
C     F -- OPTIMUM VALUE OF OBJECTIVE FUNCTION
C     ........................................................................
      COMMON /CONVEC/G
      COMMON/INCDVC/GO
      COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
      COMMON/NEW/FF,ACC,IGRAD
      COMMON/ALARM/KAL,JAW
      DIMENSION X(NN),AL(3),AF(3),Z(2)
      DIMENSION S(50),G(25),GO(25)
      KTK=1
      TT1=TT
      KAL=0
 45   CALL FUNCT(X,F)
      CALL MODIFY(F)
      Z(1)=F
      IF(NC)77,77,78
 78   DO 79 J=1,NC
 79   GO(J)=G(J)
 77   CONTINUE
 10   KAL=KAL+1
      GOTO (700,701)IGRAD
 700  CALL GRAD1(X,S,SUM,NN,F)
      GOTO 703
 701  CALL GRADU(X,S,SUM)
 703  B=SQRT(SUM)
      DO 300 J=1,NN
 300  S(J)=S(J)/B
 46   CALL RANGE(NN,NQUIT,NREP,TT,TT1,X,S,AF,AL,XH,XL,Z)
      GOTO (11,12,13)NREP
 13   GOTO (12,41,41)NQUIT
 12   GOTO (900,901)MINIM
 900  CALL GOLD(X,S,NN,XL,XH,F)
      GOTO 903
 901  CALL QUAD(X,S,NN,AL,AF,F)
 903  IF(NC)80,80,81
 81   DO 82 J=1,NC
 82   GO(J)=G(J)
 80   Z(2)=F
 11   CONTINUE
      IF(KTK-3)514,43,43
 514  IF(ABS(Z(2)-Z(1))-ACC)43,43,42
 42   Z(1)=Z(2)
      F=Z(2)
      GOTO 10
C     CONVERGENCE RULE
 43   IF(NPERT-1)41,41,44
 44   CALL CONVRG(X,S,Z,NN,NC,KTK,HK,F)
      GOTO(10,45,146,41)KTK
 146  KAL=KAL+1
      GOTO 45
C     PRINT OUTPUT
 41   F=Z(2)
      IF(IOUT.GT.4)GOTO 76
      WRITE(1,203)
 203  FORMAT('STEEPEST DESCENT METHOD USED ')
 76   IF(IOUT.GT.5)GOTO 73
      WRITE(1,176)KAL
 176  FORMAT('NO OF 1.D CYCLES ',I4)
      IF(IOUT.GT.3)GOTO 73
      WRITE(1,4) (X(J),J=1,NN)
      WRITE(1,204) Z(2),ACC
 4    FORMAT('[ X ] := ',9(1PE14.6))
 204  FORMAT('[ F ] := ', 1PE14.6//'ACCURACY ACHIEVED ',F9.6)
 73   TT=TT1
      RETURN
```

```
F1=HK-2.*FMIN(NN)+F
F2=(HK-FMIN(NN)-FM)**2
F3=.5*FM*(HK-F)**2
ZZ=F1*F2-F3
IF(ZZ)34,32,32
SUM=0.
DO 35 J=1,NN
S(J)=X(J)-XO(J)
SUM=SUM+S(J)**2
SUM=SQRT(SUM)
DO 36 J=1,NN
S(J)=S(J)/SUM
IF(NX-NN)40,41,41
NN1=NN-1
DO 37 J=1,NN1
IF(J-NX)37,38,38
DO 39 I=1,NN
SCD(J,I)=SCD(J+1,I)
CONTINUE
DO 42 J=1,NN
SCD(NN,J)=S(J)
I=NN
K=2
KT=2
KAL=KAL+1
GOTO 26
SHUFFLE THE DIRECTION MATRIX TO GET SEARCH STARTED
NNI=NN-1
GOTO (59,72,72),KT
IF(KTT-NN)44,12,12
IF(KTT-NNI)60,60,52
IF(I-NN)51,52,52
HK=F
Z(1)=F
GOTO 33
DO 45 J=1,NN
SCD(NN+1,J)=SCD(I,J)
DO 47 J=1,NN
SCD(I,J)=SCD(I+1,J)
DO 48 J=1,NNI
DO 48 JK=1,NN
SCD(I+J,JK)=SCD(I+J+1,JK)
KTT=KTT+1
GOTO 24
CHECK FOR CONVERGENCE
IF(NPERT-1)12,12,73
CALL CONVRG(X,S,Z,NN,NC,KTK,HK,F)
GOTO(77,69,76,12)KTK
DO 75 J=1,NN
XO(J)=X(J)
KAL=KAL+1
GOTO 26
DO 78 J=1,NN
DO 78 I=1,NN
SCD(I,J)=0.
DO 79 I=1,NN
DO 79 J=1,NN
IF(I-J)79,80,79
SCD(I,J)=1.
CONTINUE
KT=1
KTT=1
KTK=1
I=2
K=1
DO 82 J=1,NN
SCD(1,J)=S(J)
GOTO 24
PRINT OUTPUT
F=Z(2)
IF(IOUT.GT.4)GOTO 173
WRITE(1,58)
FORMAT('POWELL'S METHOD  [ CONDIR ] USED ')
IF(IOUT.GT.5)GOTO 55
```

```
C         ============================================================
          SUBROUTINE CONGRA(MINIM,X,TT,NN,F) !! FLETCHER-REEVES METHOD !!
C         ============================================================
C         X -- DESIGN VECTOR
C         NN -- ORDER OF THE DESIGN VECTOR
C         MINIM -- CHOICE OF 1.D SEARCH TECHNIQUE
C         MINIM=1 GOLDEN SECTION
C         MINIM=2 QUADRATIC INTERPOLATION
C         TT -- INITIAL STEP SIZE
C         ACC -- ACCURACY DESIRED
C         FF -- PERCENTAGE CHANGE IN VARIABLE DESIRED FOR FINITE
C         DIFFERENCE COMPUTATION OF THE GRADIENTS
C         IGRAD -- CHOICE OF GRADIENT ROUTINE
C         IGRAD=1 USE PACKAGE ROUTINE
C         IGRAD=2 USE USER ROUTINE
C         F -- OPTIMUM VALUE OF OBJECTIVE FUNCTION
          ............................................................
          COMMON/CONVEC/G
          COMMON/INCOVC/GO
          COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
          COMMON/NEW/FF,ACC,IGRAD
          COMMON/ALARM/KAL,JAW
          DIMENSION X(NN),AF(3),AL(3),Z(2)
          DIMENSION A(25),S(50),SO(50),G(25),GO(25)
          TT1=TT
          KTK=1
          KAL=0
          CALL FUNCT(X,F)
103       CALL MODIFY(F)
          IF(NC)77,77,78
78        DO 79 J=1,NC
79        GO(J)=G(J)
77        CONTINUE
          HK=F
3         Z(1)=F
513       KAL=KAL+1
          GOTO (700,701)IGRAD
700       CALL GRAD1(X,S,SUM,NN,F)
          GOTO 703
701       CALL GRADU(X,S,SUM)
703       IST=0
          B=SUM
          SUM=SQRT(SUM)
          DO 111 J=1,NN
          SO(J)=S(J)
111       S(J)=S(J)/SUM
48        CALL RANGE(NN,NQUIT,NREP,TT,TT1,X,S,AF,AL,XH,XL,Z)
          GOTO (51,50,52)NREP
52        GOTO (50,511,11)NQUIT
50        GOTO (900,901)MINIM
900       CALL GOLD(X,S,NN,XL,XH,F)
          GOTO 903
901       CALL QUAD(X,S,NN,AL,AF,F)
903       IF(NC)80,80,81
81        DO 82 J=1,NC
82        GO(J)=G(J)
80        CONTINUE
514       Z(2)=F
51        IF(KTK-3)53,92,92
53        IF(ABS(Z(2)-Z(1))-ACC)11,11,12
11        IF(ABS(Z(2)-HK)-ACC)92,92,512
512       HK=Z(2)
          Z(1)=Z(2)
          F=Z(2)
          GOTO 513
12        Z(1)=Z(2)
          F=Z(2)
          IF(IST-NN+1)20,20,513
20        IST=IST+1
C         DETERMINATION OF CONJUCATE GRADIENT DIRECTION.
          GOTO (705,706)IGRAD
705       CALL GRAD1(X,S,SUM,NN,F)
          GOTO 704
706       CALL GRAD(X,S,SUM)
704       BETA=SUM/B
          B=SUM
          DO 46 J=1,NN
          S(J)=S(J)+BETA*SO(J)
          SO(J)=S(J)
46        CONTINUE
          SA=0.
          DO 47 I=1,NN
47        SA=SA+S(I)**2
          DO 49 I=1,NN
49        S(I)=S(I)/SQRT(SA)
```

```
      KAL=KAL+1
      GOTO 48
C     CONVERGENCE CRITERION
92    IF(NPERT-1)511,511,516
516   CALL CONVRG(X,S,Z,NN,NC,KTK,HK,F)
      GOTO (513,103,148,511)KTK
148   KAL=KAL+1
      GOTO 48
C     PRINT OUTPUT
511   F=Z(2)
      IF(IOUT.GT.4)GOTO 76
      WRITE(1,104)
104   FORMAT('FLETCHER - POWELL METHOD (CONGRA) USED')
76    IF(IOUT.GT.5)GOTO 73
      WRITE(1,176)KAL
176   FORMAT('NO OF 1.D CYCLES ',I4)
      IF(IOUT.GT.3)GOTO 73
      WRITE(1,211)(X(I),I=1,NN)
211   FORMAT('[ X ] := ',9(1PE14.8))
      WRITE(1,205)Z(2),ACC
205   FORMAT('[ F ] := ',1PE14.6/'ACCURACY ACHIEVED ',F9.5)
73    IT=IT1
      RETURN
      END
```

```
C       ==================================================================
        SUBROUTINE DFPM(MINIM,X,TT,NN,F) !! DAVIDON-FLETCHER-POWELL'S !!
C       ==================================================================
C       X -- DESGIN VECTOR
C       NN -- ORDER OF THE DESIGN VECTOR
C       MINIM -- CHOICE OF 1.D SEARCH TECHNIQUE
C       MINIM=1 GOLDEN SECTION
C       MINIM=2 QUADRATIC INTERPOLATION
C       TT------INITIAL STEP SIZE
C       ACC-----ACCURACY DESIRED
C       FF------PERCENTAGE CHANGE IN VARIABLE DESIRED FOR FINITE
C       DIFFERENCE COMPUTATION OF THE GRADIENTS
C       IGRAD---CHOICE OF GRADIENT SUBROUTINE
C       IGRAD=1  USE PACKAGE SUBROUTINE
C       IGRAD=2 USE USER SUBROUTINE
C       F------ OPTIMUM VALUE OF OBJECTIVE FUNCTION
C       ..................................................................
        COMMON/CONVEC/G1
        COMMON/INCOVC/GO
        COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
        COMMON/MIN/AAL
        COMMON/NEW/FF,ACC,IGRAD
        COMMON/ALARM/KAL,JAW
        COMMON/PENFUN/R,F1
        DIMENSION X(NN),AL(3),AF(3),Z(2)
        DIMENSION S(50),H(25,25),FX(25),FY(25),G(25)
        DIMENSION SQ(25)
        DIMENSION GO(25),G1(25)
C       START SEARCH IN STEEPOST DESCENT DIRECTION
        TT1=TT
        KTK=1
        KAL=0
        CALL FUNCT(X,F)
103     CALL MODIFY(F)
        Z(1)=F
        IF(NC)77,77,78
78      DO 79 J=1,NC
79      GO(J)=G1(J)
77      CONTINUE
        HK=F
124     KAL=KAL+1
        DO 10 J=1,NN
        DO 10 I=1,NN
        IF(I-J)12,11,12
11      H(I,J)=1.
        GO TO 10
12      H(I,J)=0.
10      CONTINUE
        GO TO (700,701)IGRAD
700     CALL GRAD1(X,S,SUM,NN,F)
        GO TO 703
701     CALL GRADU(X,S,SUM)
703     SUM=SQRT(SUM)
        DO 13 J=1,NN
        FX(J)=-S(J)
        SQ(J)=S(J)
13      S(J)=S(J)/SUM
65      CALL RANGE(NN,NQUIT,NREP,TT,TT1,X,S,AF,AL,XH,XL,Z)
        GO TO(514,16,74)NREP
74      GO TO(16,122,121)NQUIT
16      GO TO(900,901)MINIM
900     CALL GOLD(X,S,NN,XL,XH,F)
        GOTO 903
901     CALL QUAD(X,S,NN,AL,AF,F)
903     IF(NC)80,80,81
81      DO 82 J=1,NC
82      GO(J)=G1(J)
80      Z(2)=F
C       CHECK ACCURACY AND RESTART CRITERION.
514     IF(KTK-3)515,92,92
515     IF(IOUT.GE.2)WRITE(1,95)(Z(I),I=1,2)
95      FORMAT('VALUES OF Z(1) and Z(2)'/2(1PE14.6))
        IF(ABS(Z(2)-Z(1))-ACC)121,121,22
121     IF(ABS(Z(2)-HK)-ACC)92,92,123
123     F=Z(2)
        HK=F
        Z(1)=Z(2)
        IF(IOUT.GE.2)WRITE(1,90)
90      FORMAT(/'* ITERATION IS RESTARTED FOR NEW S(J) *')
        GOTO 124
22      F=Z(2)
        GOTO (705,706)IGRAD
705     CALL GRAD1(X,S,SUM,NN,F)
        GOTO 704
C       FOR 1 VARIABLE METRIC
```

```
06    CALL GRADU(X,S,SUM)
04    DO 14 J=1,NN
4     G(J)=-S(J)-FX(J)
      SSUM=0.
      DO 54 I=1,NN
      DO 55 J=1,NN
5     SSUM=SSUM+H(I,J)*G(J)
      FY(I)=SSUM
      SSUM=0.
4     CONTINUE
      A2=0.
      DO 52 J=1,NN
2     A2=A2+SQ(J)*G(J)
      A2=AAL/A2
      DO 50 I=1,NN
      DO 50 J=1,NN
0     H(I,J)=H(I,J)+A2*SQ(I)*SQ(J)
      A2=0.
      DO 57 J=1,NN
      A2=A2+FY(J)*G(J)
      DO 56 I=1,NN
      DO 56 J=1,NN
      H(I,J)=H(I,J)-FY(I)*FY(J)/A2
      DO 53 J=1,NN
      FY(J)=-S(J)
      SSUM=0.
      DO 60 I=1,NN
      DO 61 J=1,NN
      SSUM=SSUM+H(I,J)*FY(J)
      S(I)=-SSUM
      SQ(I)=S(I)
      SSUM=0.
      CONTINUE
      DETERMINATION OF VARIABLE METRIC DIRECTION.
      DO 62 J=1,NN
      FX(J)=FY(J)
      DO 63 J=1,NN
      SSUM=SSUM+S(J)^2
      SSUM=SQRT(SSUM)
      IF(SSUM)67,66,67
      SSUM=1.
      DO 64 J=1,NN
      S(J)=S(J)/SSUM
      Z(1)=Z(2)
      KAL=KAL+1
      GOTO 65
      CONVERGENCE CRITERION
      IF(NPERT-1)122,122,516
6     CALL CONVRG(X,S,Z,NN,NC,KTK,HK,F)
      GOTO (124,103,125,122)KTK
5     KAL=KAL+1
      PRINT OUTPUT
2     F=Z(2)
      IF(IOUT.GT.4)GOTO 75
      WRITE(1,5)
      FORMAT('DAVIDON - FLETCHER - POWELL  METHOD USED')
      IF(IOUT.GT.5)GOTO 76
      WRITE(1,85)KAL
      FORMAT('NO OF 1.0 CYCLES ',I3)
      IF(IOUT.GT.3)GOTO 76
      WRITE(1,3)(X(J),J=1,NN)
      FORMAT('[ X ] := ',9(1PE14.6))
      WRITE(1,4)Z(2)
      FORMAT('[ F ] := ',1PE14.6)
      F=Z(2)
      TT=TT1
      NO=NOUIT
       RETURN
       END
```

```
C     ================================================================
      SUBROUTINE GRAD1(X,S,SUM,NN,F)
C     ================================================================
C     THIS ROUTINE DETERMINES THE DERIVATIVES USING THE FINITE
C     DIFFERENCE METHOD
C     X -- DESIGN VECETOR
C     NN -- ORDER OF THE DESIGN VECTOR
C     F -- INITIAL VALUE OF THE OBJECTIVE FUNCTION
C     SUM -- SUM OF THE SQUARES OF THE DERIVATIVES
C     S -- SEARCH DIRECTION VECTOR
C     FF -- PERCENTAGE CHANGE IN VARIABLE DESIRED FOR THE
C     COMPUTATION OF GRADIENTS
C     ................................................................
      COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
      COMMON/NEW/FF,ACC,IGRAD
      DIMENSION X(NN),S(50),SS(2)
      SS(1)=F
      SUM=0.
      DO 10 J=1,NN
      IF(X(J))60,61,60
61    Q=FF
      GOTO 62
60    Q=FF*X(J)
62    X(J)=X(J)+Q
      CALL FUNCT(X,F)
      CALL MODIFY(F)
      SS(2)=F
      X(J)=X(J)-Q
      S(J)=(SS(1)-SS(2))/Q
10    SUM=SUM+S(J)**2
      IF(SUM)11,12,11
12    SUM=1.
11    CONTINUE
      RETURN
      END


      SUBROUTINE GRAD(X,S,SUM,NN,F,NC,DG)
C     THIS ROUTINE DETERMINES THE DERIVATIVES USING THE FINITE
C     DIFFERENCE METHOD
C     X -- DESIGN VECETOR
C     NN -- ORDER OF THE DESIGN VECTOR
C     F -- INITIAL VALUE OF THE OBJECTIVE FUNCTION
C     SUM -- SUM OF THE SQUARES OF THE DERIVATIVES
C     S -- SEARCH DIRECTION VECTOR
C     FF -- PERCENTAGE CHANGE IN VARIABLE DESIRED FOR THE
C     COMPUTATION OF GRADIENTS
C     DG -- MATRIX OF CONSTRAINT GRADIENTS
C     G,GO -- CONSTRAINT VECTORS
C     NC -- NUMBER OF CONSTRAINTS
C     ................................................................
      COMMON/CONVEC/G
      COMMON/INCOVC/GO
      COMMON/NEW/FF,ACC,IGRAD
      DIMENSION X(NN),S(50),SS(2),DG(NC,NN)
      DIMENSION GO(25),G(25)
      SS(1)=F
      SUM=0.
      DO 10 J=1,NN
      IF(X(J))60,61,60
61    Q=FF
      GOTO 62
60    Q=FF*X(J)
62    X(J)=X(J)+Q
      CALL FUNCT(X,F)
      CALL MODIFY(F)
      SS(2)=F
      X(J)=X(J)-Q
      S(J)=(SS(1)-SS(2))/Q
      DO 20 K=1,NC
20    DG(K,J)=(G(K)-GO(K))/Q
10    SUM=SUM+S(J)**2
      IF(SUM)11,12,11
12    SUM=1.
11    CONTINUE
      RETURN
      END
```

```
C         ==========================================================
          SUBROUTINE MODIFY(F)
C         ==========================================================
          COMMON/PENFUN/R,UF
          COMMON/CONVEC/G
          COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIMM,NC,NI,IOUT,NPERT
          COMMON/NEW/FF,ACC,IGRAD
          DIMENSION G(25)
          IF(NC.EQ.0)RETURN
          GOTO(1,2,3)KFEAS
1         IF(NI)13,13,14
14        DO 5 J=1,NI
          IF(G(J))5,5,6
6         F=F+R*G(J)**2
5         CONTINUE
13        NE=NC-NI
          IF(NE)3,3,11
11        DO 9 J=1,NE
9         F=F+R*G(NI+J)**2
          GOTO 3
2         IF(NI)15,15,16
16        DO 7 J=1,NI
          IF(G(J))8,7,7
8         F=F-R/G(J)
7         CONTINUE
15        NE=NC-NI
          IF(NE)3,3,12
12        DO 10J=1,NE
10        F=F+G(NI+J)**2/SQRT(R)
3         RETURN
          END
```

```
C         ============================================================================
          SUBROUTINE CONSTR(NPENAL)
C         ============================================================================
C         THIS ROUTINE CHECKS FOR VIOLATION OF ANY CONSTRAINTS
C         NPENAL=1 CONSTRAINTS SATISFIED
C         NPENAL=2 CONSTRAINTS VIOLATED
C         ACC -- ACCURACY DESIRED
C         NC -- NUMBER OF CONSTRAINTS
C         N -- INDEX INDICATING NUMBER OF CONSTRAINTS VIOLATED
C         NCONST -- VECTOR GIVING INFORMATION ON CONSTRAINTS VIOLATED
C         ............................................................................
          COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
          COMMON/CONVEC/G
          COMMON/CONVIO/N,NCONST
          COMMON/NEW/FF,ACC,IGRAD
          DIMENSION G(25),NCONST(25)
          N=1
          NPENAL=1
          IF(KFEAS-1)16,15,17
17        DO 10 J=1,NI
          IF(G(J)-.1E-10)11,12,12
11        IF(ABS(G(J))-ACC)13,13,10
13        NCONST(N)=J
          GOTO 14
12        NCONST(N)=J
          NPENAL=2
14        IF(KFEAS-2)16,15,18
18        N=N+1
10        CONTINUE
          GOTO 16
15        NPENAL=2
16        RETURN
          END
```

```
C       ================================================================
        SUBROUTINE CONVRG(X,S,Z,NN,NC,KTK,ZK,F)
C       ================================================================
        COMMON/CONVEC/G
        COMMON/INCOVC/GO
        COMMON/NEW/FF,ACC,IGRAD
        DIMENSION G(25),GO(25)
        DIMENSION X(NN),S(50),Z(2),A(25)
11      GOTO (90,91,92)KTK
90      HK=Z(2)
        F=Z(2)
        Z(1)=Z(2)
        KTK=2
        DO 93 J=1,NN
93      A(J)=X(J)
        DO 94 J=1,NN
94      X(J)=1.01*X(J)
        CALL FUNCT(X,F)
        CALL CONSTR(NPENAL)
        IF(NPENAL-1)80,80,511
91      Z2HK=Z(2)-HK
        IF(ABS(Z2HK)-ACC)511,511,512
512     IF(Z(2)-HK)99,99,100
100     DO 95 J=1,NN
95      S(J)=A(J)-X(J)
        DO 83 J=1,NN
83      X(J)=A(J)
        CALL FUNCT(X,F)
        CALL MODIFY(F)
106     DO 107 J=1,NC
107     GO(J)=G(J)
105     CONTINUE
        Z(1)=HK
        GOTO 101
99      DO 102 J=1,NN
102     S(J)=X(J)-A(J)
        HK=Z(2)
        Z(1)=Z(2)
101     SUM=0.
        DO 96 J=1,NN
96      SUM=SUM+S(J)**2
        SUM=SQRT(SUM)
        DO 97 J=1,NN
97      S(J)=S(J)/SUM
        KTK=3
        GOTO 80
92      IF(ABS(Z(2)-HK)-ACC)511,511,98
98      KTK=1
        F=Z(2)
        Z(1)=Z(2)
        HK=Z(2)
        ZK=HK
        IF(NC)80,80,84
84      DO 85 J=1,NC
85      GO(J)=G(J)
        GOTO 80
511     KTK=4
80      RETURN
        END
```

```
C     ================================================================
      SUBROUTINE IPENAL(MINIM,X,TT,NN,FC,F)
C     ================================================================
C     THIS ROUTINE USES THE INTERIOR PENALTY FUNCTION TECHNIQUE
C     TO FIND THE CONSTRAINED OPTIMUM
C     X -- DESIGN VECTOR
C     NN -- ORDER OF THE DESIGN VECTOR
C     NC -- NUMBER OF CONSTRAINTS
C     TT -- INITIAL STEP SIZE
C     ACC -- ACCURACY DESIRED
C     MINIM -- CHOICE OF 1.D SEARCH TECHNIQUE
C     MINIM=1 GOLDEN SECTION
C     MINIM=2 QUADRATIC INTERPOLATION
C     METHOD -- CHOICE OF MULTIVARIATE SEARCH TECHNIQUE
C     METHOD=1 UNIVARIATE SEARCH
C     METHOD=2 STEEPEST DESCENT
C     METHOD=3 CONJUCATE DIRECTIONS
C     METHOD=4 CONJUCATE GRADIENT
C     METHOD=5 VARIABLE METRIC METHOD
C     KFEAS=1 UNCONSTRAINED OR EXTERIOR PENALTY FUNCTION PROBLEM
C     KFEAS=2 UNCONSTRAINED PROBLEM USING INTERIOR PENALTY FUNCTION
C     KFEAS=3 CONSTRAINED PROBLEM USING FEASIBLE DIRECTION METHOD
C     FF -- PERCENTAGE CHANGE IN VARIABLE DESIRED FOR FINITE
C     DIFFERENCE COMPUTATION OF THE GRADIENTS
C     R -- PENALTY FACTOR
C     F1 --UNAUGMENTED OBJECTIVE FUNCTION
C     NREP=3 TERMINATE. SEARCH DIRECTION DOESN'T IMPROVE OBJECTIVE
C     FUNCTION OR VIOLATES A CONSTRAINT
C     FC -- FACTOR FOR INCREASING OR DECREASING "R"
C     ................................................................
      COMMON/PENFUN/R,F1
      COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
      COMMON/NEW/FF,ACC,IGRAD
      COMMON/ALARM/KAL,JAW
      COMMON/CONVEC/G1
      DIMENSION W(2,25),CC(25),DD(25)
      DIMENSION V(2),RR(2),X(NN)
      DIMENSION DG(25,25),G(25),G1(25)
      JAW=1
      IF(IOUT.GT.5)GOTO 10
      WRITE(1,302)R,JAW
302   FORMAT(/40('-')/' R :=  ',1PE9.2,' CYCLE NO ',I3/40('-'))
C     PERFORM UNCONSTRAINED MINIMIZATIONS FOR TWO VALUES OF " R "
10    NR=1
15    GOTO (450,451,452,453,454)METHOD
450   CALL UNIV(MINIM,X,TT,NN,F)
      GOTO 455
451   CALL STEEP(MINIM,X,TT,NN,F)
      GOTO 455
452   CALL CONDIR(MINIM,X,TT,NN,F)
      GOTO 455
453   CALL CONGRA(MINIM,X,TT,NN,F)
      GOTO 455
454   CALL DFPM(MINIM,X,TT,NN,F)
455   IF(IOUT.LT.3.OR.IOUT.GT.5)GOTO 110
      WRITE(1,235)(X(I),I=1,NN)
      WRITE(1,76)F
235   FORMAT('[ X ] := ',9(1PE14.6))
76    FORMAT('[ F ] := ',1PE16.8)
110   CALL FUNCT(X,F)
      CALL MODIFY(F)
      F3=F
      IF(IOUT.GT.5)GOTO 35
      WRITE(1,61)F1
35    RR(NR)=R
      V(NR)=F1
      DO 40 J=1,NN
40    W(NR,J)=X(J)
      IF(NR-2)13,14,14
13    NR=NR+1
      R=FC*R
      JAW=JAW+1
      IF(IOUT.GT.5)GOTO 15
      WRITE(1,302)R,JAW
      GOTO 15
C     BEGIN THE EXTRAPOLATION SCHEME
14    IF(ABS(V(2)-V(1))-ACC)116,116,117
116   F=V(2)
      GOTO 36
117   NTAL=1
144   SC=SQRT(RR(1))
      SD=SQRT(RR(2))
      DEN=SC-SD
      IF(IOUT.GT.4)GOTO 70
80    WRITE(1,462)
```

```
462     FORMAT('EXTRAPOLATION RESULTS')
70      AA=(V(2)*SC-V(1)*SD)/DEN
        BB=(V(1)-V(2))/DEN
        DO 16 J=1,NN
        CC(J)=(W(2,J)*SC-W(1,J)*SD)/DEN
16      DD(J)=(W(1,J)-W(2,J))/DEN
        IF(IOUT.GT.4)GOTO 71
82      WRITE(1,400)
400     FORMAT('EXTRAPOLATION SCHEME IS [ f ] := AA+BB*R*.5 '/
     1         '[ f ] predicted optimum is AA ')
        WRITE(1,401) AA
401     FORMAT('AA := ',1PE16.8)
        WRITE(1,402)
402     FORMAT('EXTRAPOLATION SCHEME IS X(I)=CC(I)+DD(I)*R*.5'/
     1         '[ X ] predicted optimum design vector is CC(I)')
        WRITE(1,403)(CC(K),K=1,NN)
403     FORMAT(9(1PE16.8))
71      IF(NTAL-1)25,25,26
25      R=FC*R
        JAW=JAW+1
        AAO=AA
        FMIN=AA+BB*SQRT(R)
        DO 17 J=1,NN
17      DD(J)=CC(J)+DD(J)*SQRT(R)
        CALL FUNCT(DD,F)
        CALL MODIFY(F)
        F2=F
        CALL CONSTR(NPENAL)
        IF(NPENAL-1)56,56,85
56      IF(F3-F2)57,57,53
53      DO 58 J=1,NN
58      X(J)=DD(J)
        IF(IOUT.GT.4)GOTO 57
        WRITE(1,86)R
86      FORMAT('EXTRAPOLATED VECTOR VALID WITH NEW R ',1PE9.2)
        GOTO 57
85      CALL FUNCT(CC,F)
        CALL MODIFY(F)
        F4=F
        CALL CONSTR(NPENAL)
        IF(NPENAL-1)59,59,90
90      IF(IOUT.GT.4)GOTO 57
        WRITE(1,93)
93      FORMAT('* PREDICTED VECTORS INFEASIBLE *')
        GOTO 57
59      IF(F3-F4)88,88,91
91      DO 92 J=1,NN
92      X(J)=CC(J)
        WRITE(1,87)
87      FORMAT('PREDICTED VECTOR CC VALID')
        GOTO 57
88      WRITE(1,89)
89      FORMAT('EXTRAPOLATED and PREDICTED OPTIMUM VECTORS INCREASE
     1         THE FUNCTION SO UNSUITABLE')
57      IF(IOUT.GT.5)GOTO 72
        WRITE(1,405)
405     FORMAT('NEW STARTING POINT')
        WRITE(1,403)(X(J),J=1,NN)
        WRITE(1,302)R,JAW
C       PERFORM UNCONSTRAINED MINIMIZATION FOR NEW VALUE OF "R"
72      GOTO (456,457,458,459,460)METHOD
456     CALL UNIV(MINIM,X,TT,NN,F)
        GOTO 461
457     CALL STEEP(MINIM,X,TT,NN,F)
        GOTO 461
458     CALL CONDIR(MINIM,X,TT,NN,F)
        GOTO 461
459     CALL CONGRA(MINIM,X,TT,NN,F)
        GOTO 461
460     CALL DFPM(MINIM,X,TT,NN,F)
461     IF(IOUT.LT.3.OR.IOUT.GT.5)GOTO 111
        WRITE(1,235)(X(I),I=1,NN)
        WRITE(1,75)F
111     CALL FUNCT(X,F)
        CALL MODIFY(F)
        F3=F
        IF(IOUT.GT.5)GOTO 73
        WRITE(1,61)F1
61      FORMAT('[ f ] unaugmented := ',1PE16.8)
73      IF(ABS(V(2)-F1)-ACC)33,33,37
C       CHECK ACCURACY CRITERION, IF NOT SATISFIED RE-COMPUTE CONSTANTS
37      IF(ABS(F1-FMIN)-ACC)18,18,19
18      NTAL=NTAL+1
19      RR(1)=RR(2)
        RR(2)=R
```

```
          V(1)=V(2)
          V(1)=V(2)
          V(2)=F1
          DO 54 J=1,NN
          W(1,J)=W(2,J)
54        W(2,J)=X(J)
          GOTO 144
26        IF(ABS(F1-AAD)-ACC)33,33,31
31        NTAB=1
          GOTO 25
33        F=F1
36        IF(IOUT.GT.6)GOTO 74
          WRITE(1,235)(X(I),I=1,NN)
          WRITE(1,52)F,JAW,ACC
52        FORMAT('[ F ] := ',1PE16.8//'NO OF CONSTRAINED CYCLES',I3
1              /'ACCURACY ACHIEVED  := ',1PE8.2)
74        RETURN
          END
```

```
C     ==================================================================
      SUBROUTINE EXPEN(MINIM,X,TT,NN,FC,F)
C     ==================================================================
C     THIS ROUTINE DETERMINES THE CONSTRAINED OPTIMUM USING THE
C     EXTERIOR PENALTY FUNCTION TECHNIQUE
C     X -- DESIGN VECTOR
C     NN -- ORDER OF THE DESIGN VECTOR
C     MINIM -- CHOICE OF THE DESIGN VECTOR
C     MINIM=1 GOLDEN SECTION
C     MINIM=2 QUADRATIC INTERPOLATION
C     TT -- INITIAL STEP SIZE
C     NC -- NUMBER OF CONSTRAINTS
C     ACC -- ACCURACY DESIRED
C     FF -- PERCENTAGE CHANGE IN VARIABLE DESIRED FOR FINITE
C     DIFFERENCE COMPUTATION OF GRADIENTS
C     IGRAD -- CHOICE OF GRADIENT ROUTINE
C     IGRAD=1 USE PACKAGE ROUTINE
C     IGRAD=2 USE USER ROUTINE
C     METHOD -- CHOICE OF MULTIVARIATE SEARCH TECHNIQUE
C     METHOD=1 UNIVARIATE SEARCH
C     METHOD=2 STEEPEST DESCENT
C     METHOD=3 CONJUCATE DIRECTIONS
C     METHOD=4 CONJUCATE GRADIENT
C     METHOD=5 VARIABLE METRIC METHOD
C     KFEAS=1 UNCONDTRAINED OR EXTERIOR PENALTY FUNCTION PROBLEM
C     KFEAS=2 UNCONSTRAINED PROBLEM USING INTERIOR PENALTY FUNCTION
C     KFEAS=3 CONSTRAINED PROBLEM USING FEASIBLE DIRECTION METHOD
C     R -- PENALTY FACTOR
C     F1 -- UNAUGMENTD OBJECTIVE FUNCTION
C     FC -- FACTOR FOR INCREASING OR DECREASING " R "
C     ..................................................................
      COMMON/PENFUN/R,F1
      COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
      COMMON/NEW/FF,ACC,IGRAD
      COMMON/ALARM/KAL,JAW
      DIMENSION X(NN),CC(2)
      J=1;JAW=1
      IF(IOUT.GT.5)GOTO 3
      WRITE(1,10)R,JAW
C     PERFORM UNCONSTRAINED MINIMIZATIONS FOR TWO VALUES OF R
3     GOTO (450,451,452,453,454),METHOD
450   CALL UNIV(MINIM,X,TT,NN,F)
      GOTO 455
451   CALL STEEP(MINIM,X,TT,NN,F)
      GOTO 455
452   CALL CONDIR(MINIM,X,TT,NN,F)
      GOTO 455
453   CALL CONGRA(MINIM,X,TT,NN,F)
      GOTO 455
454   CALL DFPM(MINIM,X,TT,NN,F)
455   IF(IOUT.GT.5)GOTO 110
      WRITE(1,7)(X(I),I=1,NN)
      WRITE(1,235)F
7     FORMAT('[ X ] := ',9(1PE14.6))
235   FORMAT('[ F ] := ',1PE14.6)
110   CALL FUNCT(X,F)
      CALL MODIFY(F)
      CC(J)=F1
      IF(IOUT.GT.5)GOTO 1
      WRITE(1,61)CC(J)
61    FORMAT('[ F ] unaugmented := ',1PE16.8)
1     IF(J-2)11,12,12
11    R=FC*R
      JAW=JAW+1
      J=2
      IF(IOUT.GT.5)GOTO 3
      WRITE(1,10)R,JAW
      GOTO 3
C     CHECK ACCURACY CRITERION. IF NOT SATISFIED PERFORM UNCONSTRAINED
C     MINIMIZATION FOR NEW VALUE OF R
12    IF(ABS(CC(1)-CC(2))-ACC)4,4,5
5     CC(1)=CC(2)
      R=FC*R;JAW=JAW+1
      IF(IOUT.GT.5)GOTO 3
      WRITE(1,10)R,JAW
10    FORMAT(40('-')/'R := ',1PE9.2,' CYCLE NO ',I2,/40('-'))
      GOTO 3
4     F=CC(2)
      IF(IOUT.GT.6)GOTO 2
      WRITE(1,7)(X(J),J=1,NN)
      WRITE(1,8)CC(2),JAW,ACC
8     FORMAT('[ F ] := ',1PE16.8//'NO OF CONSTRAINED CYCLES ',I3/
     1       /'ACCURACY ACHIEVED ',1PE8.2)
2      RETURN
       END
```

```
C     ==================================================================
      SUBROUTINE FEAS(MINIM,X,TT,NN,W,NC,DG)
C     ==================================================================
C     THIS ROUTINE DETERMINES THE CONSTRAINED OPTIMUM USING
C     THE METHOD OF FEASIBLE DIRECTIONS
C     X -- DESIGN VECTOR
C     NN -- ORDER OF THE DESIGN VECTOR
C     MINIM -- CHOICE OF 1.D SEARCH TECHNIQUE
C     MINIM=1 GOLDEN SECTION
C     MINIM=2 QUADRATIC INTERPOLATION
C     IGRAD -- CHOICE OF GRADIENT ROUTINE
C     IGRAD=1 USE PACKAGE ROUTINE
C     IGRAD=2 USE USER ROUTINE
C     TT -- INITIAL STEP SIZE
C     ACC -- ACCURACY DESIRED
C     FF -- PERCENTAGE CHANGE IN VARIABLE DESIRED FOR FINITE
C     DIFFERENCE COMPUTATION OF GRADIENTS
C     W -- VECTOR OF PUSH - OFF FACTORS
C     NC -- NUMBER OF CONSTRAINTS
C     N -- INDEX INDICATING NUMBER OF CONSTRAINTS VIOLATED
C     NCONST -- VECTOR GIVING INFO ON CONSTRAINTS VIOLATED
C     DG -- MATRIX OF CONSTRAINT GRADIENTS
C     G,GO -- CONSTRAINT VECTORS
C     ..................................................................
      COMMON/CONVIO/N,NCONST
      COMMON/CONVEC/G
      COMMON/INCOVC/GO
      COMMON/NEW/FF,ACC,IGRAD
      DIMENSION X(NN),Z(2),AL(3),AF(3),W(NC),DG(NC,NN)
      DIMENSION S(50),G(25),GO(25),NCONST(25)
      DIMENSION A(25,50),B(25),C(50)
      KJ=1
      CALL FUNCT(X,F)
      Z(1)=F
      DO 70 J=1,NC
70    GO(J)=G(J)
      TT1=TT
23    GOTO (700,701)IGRAD
700   CALL GRAD(X,S,SUM,NN,F,NC,DG)
      GOTO 703
701   CALL GRADU(X,S,SUM)
703   SUM=SQRT(SUM)
      DO 111 J=1,NN
111   S(J)=S(J)/SUM
24    CALL RANGE(NN,NQUIT,NREP,TT,TT1,X,S,AF,AL,XH,XL,Z)
      GOTO (10,11,12)NREP
11    GOTO (900,901)MINIM
900   CALL GOLD(X,S,NN,XL,XH,F)
      GOTO 903
901   CALL QUAD(X,S,NN,AL,AF,F)
903   DO 40 J=1,NC
40    GO(J)=G(J)
      Z(2)=F
10    IF(ABS(Z(2)-Z(1))-ACC)122,122,22
22    Z(1)=Z(2)
      F=Z(2)
      GOTO 23
12    F=Z(2)
      GOTO (122,13,122)NQUIT
13    IF(KJ-2)80,81,81
80    F1=Z(2)
      KJ=2
      GOTO 82
81    F2=Z(2)
      IF(ABS(F2-F1)-ACC)122,122,83
83    F1=F2
82    IX=N-1
C     CONSTRAINTS IN EFFECT. COMPUTE USABLE FEASIBLE DIRECTION
C     USING LINEAR PROGRAMMING
      N1=N+NN
      N2=NN+1+N1
      N3=NN+1
      DO 79 I=1,N1
      DO 79 J=1,N2
79    A(I,J)=0.
      DO 31 I=1,NN
      DO 31 J=1,NN
      IF(I-J)31,33,31
33    A(IX+1+I,J)=1.
31    CONTINUE
      DO 35 J=1,NN
35    B(IX+1+J)=2.
      GOTO (705,706),IGRAD
705   CALL GRAD(X,S,SUM,NN,F,NC,DG)
      GOTO 704
```

```
706   CALL GRADU(X,S,SUM)
704   DO 50 J=1,IX
      K=NCONST(J)
      DO 51 I=1,NN
51    A(J,I)=DG(K,I)
      SUM=0.
      DO 52 I=1,NN
52    SUM=SUM+DG(K,I)
      B(J)=SUM
50    CONTINUE
      DO 55 J=1,IX
      K=NCONST(J)
      A(J,N3)=W(K)
56    CONTINUE
      IX=IX+1
      A(IX,N3)=1.
      DO 54 I=1,N1
      DO 54 J=1,N1
      IF(I-J)54,57,54
57    A(I,N3+J)=1.
54    CONTINUE
      DO 74 J=1,N2
74    C(J)=0.
      C(N3)=-1.
      SUM=0.
      DO 30 J=1,NN
      A(IX,J)=-S(J)
30    SUM=SUM-S(J)
      B(IX)=SUM
      CALL LINEAR(A,B,C,S,N1,N2,N3)
      DO 78 J=1,NN
78    S(J)=S(J)-1.
      SUM=0.
      DO 59 J=1,NN
59    SUM=SUM+S(J)**2
      SUM=SQRT(SUM)
      IF(SUM)61,62,61
62    SUM=1.
61    DO 60 J=1,NN
60    S(J)=S(J)/SUM
      GOTO 24
122   WRITE(1,5)(X(J),J=1,NN)
5     FORMAT('[ X ] := ',9(1PE14.6))
      WRITE(1,4)Z(2)
4     FORMAT('[ F ] := ',1PE14.6)
73    RETURN
      END
```

```
C       ======================================================================
        SUBROUTINE LINEAR(A,B,C,Z,N1,N2,N3)
C       ======================================================================
C       SOLUTION OF A L.P PROBLEM BY SIMPLEX METHOD
C       A -- COEFFICIENT MATRIX
C       B -- VECTOR OF QUANTITIES ON THE RIGHT SIDE OF THE EQUATIONS
C       C -- SENSITIVITY COEFFICIENTS
C       N1 -- NUMBER OF INEQUALITIES
C       N2 -- TOTAL NUMBER OF VARIABLES:= REAL + SLACK
C       N3 -- NUMBER OF REAL VARIABLES
C       .......................................................................
        COMMON/ALWAYS/TMAX,METHOD,KFEAS,LIM,NC,NI,IOUT,NPERT
        DIMENSION A(25,50),B(25),C(50),E(25),CB(50),Z(50),NBASIS(25)
        IF(IOUT-1)101,101,102
101     WRITE(1,95)
95      FORMAT(' LINEAR PROGRAMMING RESULTS')
        WRITE(1,66)N1,N2,N3
66      FORMAT(' INEQUALITIES =',I4,'NOS. OF VARIABLES =',I4,
       1'REAL VARIABLES =',I4)
C       FORM INITIAL BASIS
102     DO 10 J=1,N1
        NBASIS(J)=N3+J
10      CB(J)=C(N3+J)
        IF(IOUT-1)97,97,87
97      WRITE(1,60)
60      FORMAT(' NBASIS',5X,'CB',8X,'B',60X,'V')
        DO 94 J=1,N1
        WRITE(1,61)NBASIS(J),CB(J),B(J),(A(J,K),K=1,N2)
94      CONTINUE
C       CHECK IF ALL B ,S ARE POSITIVE. IF NOT RESHUFFLE BASIS.
87      DO 74 J=1,N1
        IF(B(J))73,74,74
73      IF(ABS(B(J))-1.E-10)89,89,90
89      B(J)=0.
        GOTO 74
90      DO 72 K=1,N2
72      A(J,K)=-A(J,K)
        B(J)=-B(J)
        M=1
        DO 76 K=1,N2
        IF(A(J,K))91,91,77
77      IF(A(J,K)-1.E-10)91,91,92
91      M=M+1
76      CONTINUE
92      IF(M-N2)93,93,47
93      D=A(J,K)
        MC=K
        DO 78 K=1,N2
        IF(A(J,K))78,78,79
79      IF(D-A(J,K))80,78,78
80      D=A(J,K)
        MC=K
78      CONTINUE
        NBASIS(J)=MC
        CB(J)=C(MC)
        DO 85 K=1,N2
85      A(J,K)=A(J,K)/D
        B(J)=B(J)/D
        DO 81 K=1,N1
        IF(K-J)82,81,82
82      D=A(K,MC)
        DO 83 I=1,N2
83      A(K,I)=A(K,I)-D*A(J,I)
        B(K)=B(K)-D*B(J)
81      CONTINUE
74      CONTINUE
        DO 86 J=1,N1
        IF(B(J))87,86,86
86      CONTINUE
        SUM=0.
        DO 56 J=1,N1
C       SELECT COLUMN TO BE INCLUDED IN THE BASIS
56      SUM=SUM+B(J)*CB(J)
        F=SUM
        DO 12 K=1,N2
        SUM=0.
        DO 11 J=1,N1
11      SUM=SUM+A(J,K)*CB(J)
        Z(K)=SUM-C(K)
12      CONTINUE
        KM=1
24      N=1
        DO 40 K=1,N2
        IF(Z(K)-1.E-10)44,44,41
44      N=N+1
```

```
40        CONTINUE
41        IF(N-N2)42,42,43
42        D=Z(K)
          MC=K
          DO 13 J=1,N2
          IF(Z(J))13,13,30
30        IF(D-Z(J))14,13,13
14        D=Z(J)
          MC=J
13        CONTINUE
C         SELECT ROW TO BE REMOVED FROM THE BASIS.
          DO 15 J=1,N1
          IF(A(J,MC))71,71,96
96        IF(A(J,MC)-1.E-10)71,71,70
70        E(J)=B(J)/A(J,MC)
          GOTO 15
71        E(J)=-1.
15        CONTINUE
          M=1
          DO 27 J=1,N1
          IF(E(J))45,28,28
45        M=M+1
27        CONTINUE
28        IF(M-N1)46,46,47
46        D=E(J)
          MR=J
          DO 16 J=1,N1
          IF(E(J))16,31,31
31        IF(D-E(J))16,16,17
17        D=E(J)
          MR=J
16        CONTINUE
          IF(KM-1)5,5,6
5         IF(IOUT-1)98,98,6
98        WRITE(1,60)
          DO 64 J=1,N1
          WRITE(1,61)NBASIS(J),CB(J),B(J),(A(J,K),K=1,N2)
61        FORMAT(2X,I4,3(10F10.4,/,26X))
64        CONTINUE
          WRITE(1,63)F,(Z(K),K=1,N2)
63        FORMAT(2X,'F = ',F10.4,/2X,'Z(K) =',3(12F10.4,/,7X))
C         FORMATION OF THE NEW SIMPLEX TABLE BY REPLACING ROW IN THE BASIS
C         BY THE COULUMN. REPEAT THE CYCLE OF OPERATIONS.
6         CB(MR)=C(MC)
          NBASIS(MR)=MC
          Q=Z(MC)
          DO 23 K=1,N2
23        Z(K)=Z(K)-A(MR,K)*Q/A(MR,MC)
          D=A(MR,MC)
          DO 18 J=1,N2
18        A(MR,J)=A(MR,J)/D
          B(MR)=B(MR)/D
          DO 19 J=1,N1
          IF(J-MR)21,19,21
21        D=A(J,MC)
          DO 20 K=1,N2
20        A(J,K)=A(J,K)-D*A(MR,K)
          B(J)=B(J)-D*B(MR)
19        CONTINUE
          SUM=0.
          DO 55 J=1,N1
55        SUM=SUM+B(J)*CB(J)
          F=SUM
          IF(IOUT-1)99,99,100
99        WRITE(1,60)
          DO 75 J=1,N1
          WRITE(1,61)NBASIS(J),CB(J),B(J),(A(J,K),K=1,N2)
75        CONTINUE
          WRITE(1,63)F,(Z(K),K=1,N2)
100       KM=2
          GOTO 24
C         PRINT OUTPUT
43        SUM=0.
          DO 22 J=1,N1
22        SUM=SUM+B(J)*CB(J)
          F=SUM
          DO 50 K=1,N2
50        Z(K)=0.
          DO 51 J=1,N1
          K=NBASIS(J)
          Z(K)=B(J)
51        CONTINUE
          IF(IOUT-4)103,103,52
103       WRITE(1,1)(Z(J),J=1,N3)
1         FORMAT('[ X ] := ',9(1PE14.6))
```

```
      WRITE(1,3)F
3     FORMAT('[ F ] := ',1PE14.6)
      GOTO 52
47    WRITE(1,4)
4     FORMAT(' NO SOLUTION POSSIBLE')
52    CONTINUE
      RETURN
      END
```

```
C       ================================================================
        SUBROUTINE START(X,NN,NC,TT,DG,IOUT)=============================
C       ================================================================
C       DETERMINES A FEASIBLE STARTIMG POINT
C       X -- DESIGN VECTOR
C       NN -- ORDER OF THE DESIGN VECTOR
C       IGRAD -- CHOICE OF GRADIENT ROUTINES
C       IGRAD=1 USE PACKAGE ROUTINE
C       IGRAD=2 USE USER ROUTINE
C       TT -- INITIAL STEP SIZE
C       FF -- PERCENTAGE CHANGE IN VARIABLE DESIRED FOR FINITE
C       DIFFERENCE COMPUTATION OF THE GRADIENTS
C       NC -- NUMBER OF CONSTRAINTS
C       G -- CONSTRAINT VECTOR
C       DG -- MATRIX OF CONSTRAINT GRADIENTS
C       ................................................................
        COMMON/CONVEC/G
        COMMON/INCOVC/GO
        COMMON/NEW/FF,ACC,IGRAD
        DIMENSION X(NN),DG(NC,NN)
        DIMENSION S(50),G(25),YX(25),GO(25)
        DOUBLE PRECISION G,GO,D,DG,SUM,S,X,YX,AAL,AALO
C       COMPUTE CONSTRAINTS. DETERMINE CONSTRAINT MOST VIOLATED
        CALL FUNCT(X,F)
        IF(IOUT.GE.2)GOTO 13
90      WRITE(1,19)
        WRITE(1,17)(X(J),J=1,NN)
        WRITE(1,18)
        WRITE(1,17)(G(J),J=1,NC)
13      N=1
        DO 24 J=1,NC
24      GO(J)=G(J)
        AAL=TT
        DO 5 K=1,NC
        IF(G(K))1,2,2
1       N=N+1
5       CONTINUE
2       IF(N-NC)7,7,10
7       D=G(K)
        MC=K
        DO 3 K=1,NC
        IF(G(K)-D)3,3,6
6       D=G(K)
        MC=K
3       CONTINUE
C       DETERMINE SEARCH DIRECTION. NORMAL TO CONSTRAINT MOST VIOLATED
28      GOTO (700,701)IGRAD
700     CALL GRAD(X,S,SUM,NN,F,NC,DG)
        GOTO 703
701     CALL GRADU(X,S,SUM)
703     DO 11 J=1,NN
11      S(J)=-DG(MC,J)
        SUM=0.
        DO 20 J=1,NN
20      SUM=SUM+S(J)*2
        SUM=SQRT(SUM)
        IF(SUM)21,22,21
22      SUM=1.
21      CONTINUE
        DO 23 J=1,NN
23      S(J)=S(J)/SUM
        IF(IOUT.GE.2)GOTO 15
91      WRITE(1,35)
35      FORMAT('VALUES OF S')
        WRITE(1,17)(S(J),J=1,NN)
15      DO 12 J=1,NN
12      YX(J)=X(J)+AAL*S(J)
        CALL FUNCT(YX,F)
        IF(IOUT.GE.2)GOTO 93
92      WRITE(1,19)
        WRITE(1,17)(YX(J),J=1,NN)
        WRITE(1,18)
        WRITE(1,17)(G(J),J=1,NC)
        WRITE(1,79)
79      FORMAT('VALUE OF G(MC) and GO(MC)')
        WRITE(1,17)G(MC),GO(MC)
93      IF(G(MC)-GO(MC))27,27,30
30      DO 31 J=1,NN
31      YX(J)=X(J)+AALO*S(J)
        DO 9 J=1,NN
9       X(J)=YX(J)
        GOTO 28
27      DO 29 J=1,NC
29      GO(J)=G(J)
        AALO=AAL
```

```
C       CHECK IF ALL CONSTRAINTS ARE SATISFIED. IF NOT REPEAT THE CYCLE)
        IF(G(MC))32,14,14
32      DO 33 J=1,NN
33      X(J)=YX(J)
        GOTO 13
14      AAL=2.*AAL
        GOTO 15
10      IF(IOUT.GE.2)GOTO 95
94      WRITE(1,16)
16      FORMAT('FEASIBLE STARTING POINT')
        WRITE(1,19)
19      FORMAT('VALUES OF X ARE')
        WRITE(1,17)(X(J),J=1,NN)
17      FORMAT(9(1PE14.6))
        WRITE(1,18)
18      FORMAT('VALUES OF G')
        WRITE(1,17)(G(J),J=1,NC)
95      RETURN
        END
```